



Modelling the thermodynamic equilibrium of struvite precipitation using a hybrid optimization technique

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AGENCIA NACIONAL
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NUTRIENTS IN WASTEWATER

“

Agro-industrial wastewater streams, such as concentrated slaughterhouse wastewater, present high concentrations of nutrients that generally exceed those accepted for the discharge standards

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NEGATIVE IMPACTS OF NUTRIENT DISCHARGED

01

Eutrophication



Excessive richness of nutrients in bodies of water, causes a dense growth of plant life and death of animal life from lack of oxygen.

NEGATIVE IMPACTS OF NUTRIENT DISCHARGED

02

Crystalline deposits by uncontrolled deposition of phosphate salts in wastewater treatment systems

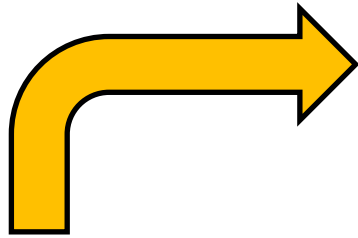


[1] *Bowers, 2011.*

NEGATIVE IMPACTS OF NUTRIENT DISCHARGED

03

Phosphorus: an increasingly limited resource



"Phosphorus is the limiting resource, the bottleneck of agriculture and therefore of the global food security"



Main models developed estimate that the duration of P reserves will be **between 50 and 100 years.**

Consumption in 2015: 43.7 million MT [2]

[2] U.S. Geological Survey (2016)

Thus it is necessary to consider appropriate treatments that reduce the concentration of nutrients, seeking to ensure economic, social and environmental sustainability of these activities.

An alternative solution that allows the recovery of nutrients in wastewater is the crystallization as struvite.

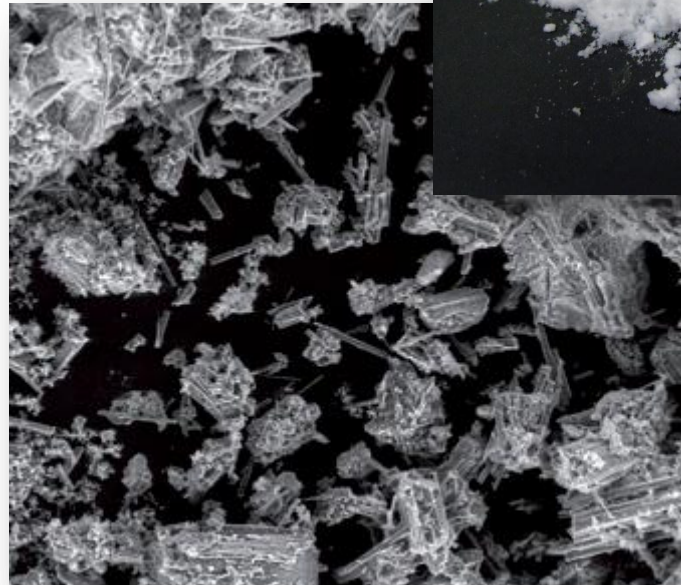
STRUVITE PRECIPITATION

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Precipitation of ***struvite*** can be conceived as one of the main processes for recovery P and N from agro-industrial wastewater streams

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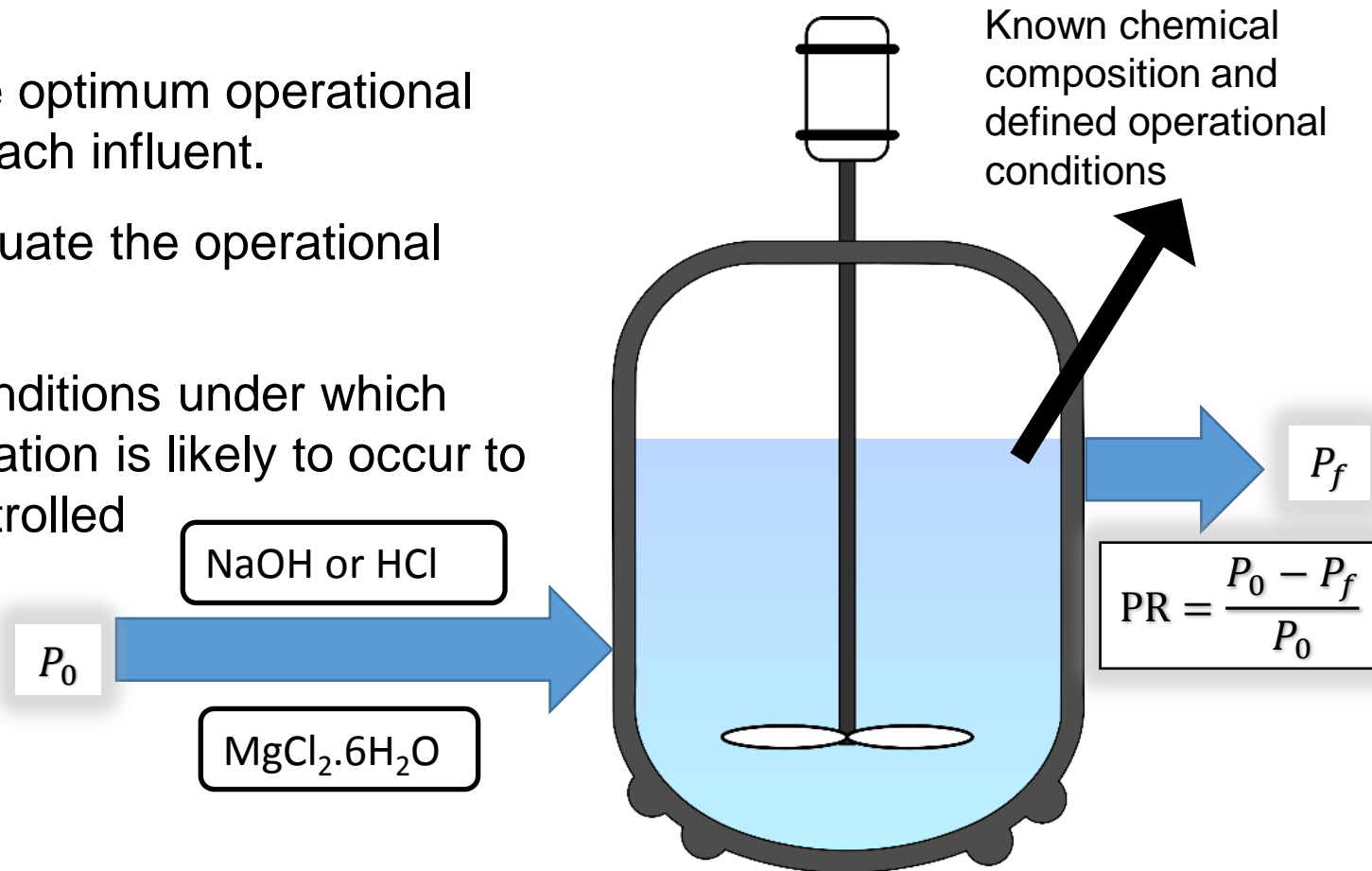
Struvite crystals



Magnesium
ammonium phosphate
hexahydrate:
 $\text{MgNH}_4\text{PO}_4 \cdot 6\text{H}_2\text{O}$

PREDICTION OF POTENTIAL NUTRIENT REMOVAL FROM WASTEWATER STREAM

- Essential tool for design crystallization reactors.
- Definition of the optimum operational conditions for each influent.
- It allows to evaluate the operational efficiency.
- To know the conditions under which struvite precipitation is likely to occur to avoid its uncontrolled deposition.



STRUVITE PRECIPITATION THERMODYNAMIC

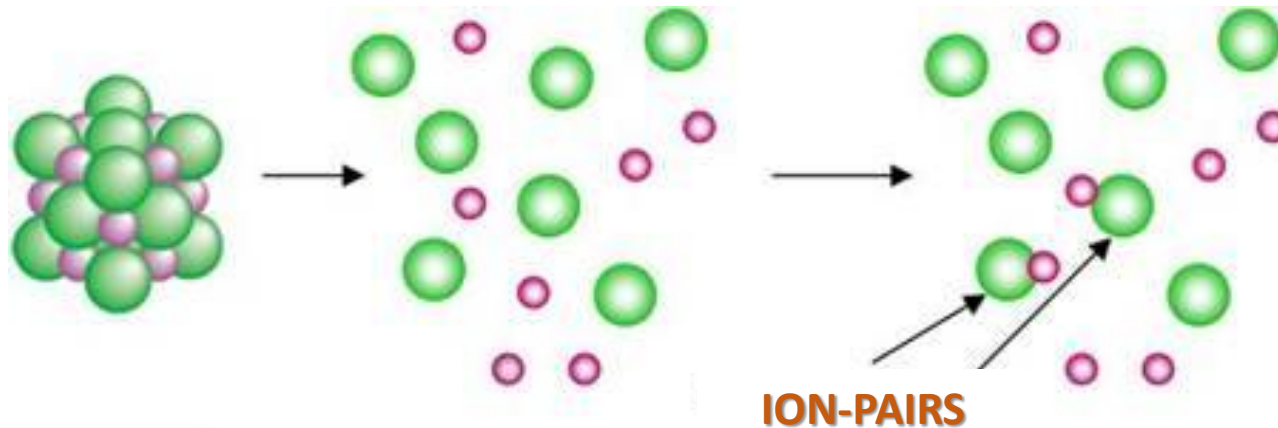
- Mg^{2+} , NH_4^+ and PO_4^{3-} concentrations
- pH
- Supersaturation ratio
- Ionic strength
- Temperature
- Reaction time



$$K_{\text{PS}} = a_{\text{Mg}^{2+}} \cdot a_{\text{NH}_4^+} \cdot a_{\text{PO}_4^{3-}}$$

Struvite solubility product

STRUVITE PRECIPITATION THERMODYNAMIC



$$a_i = \gamma_i \cdot [C_i]$$

Lewis and Randall Equation:

$$I = 0.5 \sum C_i Z_i^2$$

Calculated with Davies Equation:

$$\log \gamma_i = -A_{DH} Z_i^2 \left[\left(\frac{I^{0.5}}{1 + I^{0.5}} \right) - 0.3I \right]$$

Debye-Hückel constant:

$$A_{DH} = 1.82 \cdot 10^6 (\epsilon T)^{3/2}$$

STRUVITE PRECIPITATION THERMODYNAMIC

Supersaturation of the solution is the key parameter leading to crystallization

The supersaturation ratio (S_c) is calculated using the equation:

$$S_c = \left(\frac{P_S}{P_{Seq}} \right)^{1/3}$$

Product of analytical molar concentration

Equilibrium conditional solubility product

$S_c > 1.0$ indicates that supersaturated conditions exist and that **precipitation is possible**

$S_c = 1$ characterizes the **saturated condition**

$S_c < 1$ indicates an **undersaturated solution**

MODEL FORMULATION

A struvite precipitation model at least requires the incorporation of concentrations of :

- **Ionic species:** Mg^{2+} , NH_4^+ and PO_4^{3-}
- **Dissolved species:** NH_3 (aq) and H_3PO_4
- **Solid compound:** $\text{MgNH}_4\text{PO}_4 \cdot 6\text{H}_2\text{O}$

The complexity of the models depends on the number of soluble and solid species considered.

- **Ionic species:** H_2PO_4^- , HPO_4^{2-} , $\text{MgH}_2\text{PO}_4^+$, MgOH^+ , MgPO_4^-
- **Dissolved species:** NH_3 (gas), MgHPO_4
- **Solid compounds:** $\text{Mg}(\text{OH})_2$ (brucite), $\text{Mg}_3(\text{PO}_4)_2 \cdot 8\text{H}_2\text{O}$ (bobierrite), $\text{Mg}_3(\text{PO}_4)_2 \cdot 22\text{H}_2\text{O}$ (cattiite), and $\text{MgHPO}_4 \cdot 3\text{H}_2\text{O}$ (newberyite)

MODEL FORMULATION

This research considers the equations for ions, dissolved and solids species given in the table:

| j | Reaction j | pK_j |
|-----|---|--------|
| (1) | $\text{H}_3\text{PO}_4 \rightleftharpoons \text{H}_2\text{PO}_4^- + \text{H}^+$ | 2.15 |
| (2) | $\text{H}_2\text{PO}_4^- \rightleftharpoons \text{HPO}_4^{2-} + \text{H}^+$ | 7.21 |
| (3) | $\text{HPO}_4^{2-} \rightleftharpoons \text{PO}_4^{3-} + \text{H}^+$ | 12.34 |
| (4) | $\text{MgH}_2\text{PO}_4^+ \rightleftharpoons \text{H}_2\text{PO}_4^- + \text{Mg}^{2+}$ | 1.51 |
| (5) | $\text{MgHPO}_4 \rightleftharpoons \text{HPO}_4^{2-} + \text{Mg}^{2+}$ | 2.91 |
| (6) | $\text{MgPO}_4^- \rightleftharpoons \text{PO}_4^{3-} + \text{Mg}^{2+}$ | 6.59 |
| (7) | $\text{MgOH}^+ \rightleftharpoons \text{Mg}^{2+} + \text{OH}^-$ | 2.56 |
| (8) | $\text{NH}_4^+ \rightleftharpoons \text{NH}_{3(\text{aq})} + \text{H}^+$ | 9.25 |
| (9) | $\text{H}_2\text{O} \rightleftharpoons \text{OH}^- + \text{H}^+$ | 14.00 |

$$pK_j = -\log e^{\Delta G^\circ / (RT)}$$



MODEL ASSUMPTIONS

The thermodynamic model is formulated with the following assumptions:

1. **Precipitate** which may be formed in assay recovery conditions is **only struvite**. Precipitates such as **cattiite** and **newberyite** were **not taken into account**. The presence of **Mg(OH)₂** in the system, which compete with struvite formation at pH>10 **is neglected**.
2. All reactions are in equilibrium state. The **reactions are rapid**; therefore, **the dynamics of the reactions are ignored** and equilibrium relationships are used to determine the species concentrations.

MODEL ASSUMPTIONS

The thermodynamic model is formulated with the following assumptions:

3. The system is run at isothermal and isobaric conditions: **25°C** and **101.325 kPa**.
4. **Non-reactive ionic species are represented by NaCl** to study the impact of ionic strength in the phosphorus removal.
5. **pH** is kept constant (**6.9 - 10.0**) by addition of NaOH or HCl, which meant that the concentration of protons and OH⁻ are known.
6. The formation of **complexes with Cl⁻ or Na⁺** is **neglected**, just like complexes created since more than two ions.

MODEL FORMULATION

In a reaction system the equilibrium condition at constant temperature and pressure occurs when the Gibbs Free Energy (G) reaches a minimum

$$G = \sum_{i=1}^N \sum_{k=1}^{\pi} C_{ik} \mu_{ik}$$



Objective Function

$$\begin{aligned} \mu_{iL} &= \mu_{iL}^{\circ}(T) + RT \ln(C_i \gamma_i) \\ \mu_{iS} &= \mu_{iS}^{\circ}(T) \end{aligned}$$

MASS AND ELECTRONEUTRALITY BALANCES

Mass balance for Mg:

$$C_{T,Mg} = C_{MgH_2PO_4^+} + C_{Mg^{2+}} + C_{MgHPO_4} + C_{MgPO_4^-} + C_{MgOH^+} + PR \cdot C_{T,OP}$$

Mass balance for orthophosphate phosphorus:

$$C_{T,OP} = C_{H_3PO_4} + C_{H_2PO_4^-} + C_{HPO_4^{2-}} + C_{PO_4^{3-}} + C_{MgPO_4^-} + C_{MgH_2PO_4^+} + C_{MgHPO_4} + PR \cdot C_{T,OP}$$

Mass balance for ammonium:

$$C_{T,NH_4} = C_{NH_4^+} + C_{NH_3} + PR \cdot C_{T,OP}$$

Electroneutrality:

$$C_{Na^+} + C_{MgH_2PO_4^+} + 2 C_{Mg^{2+}} + C_{MgOH^+} + C_{H^+} + C_{NH_4^+} - C_{MgPO_4^-} - 3C_{PO_4^{3-}} - C_{H_2PO_4^-} - C_{OH^-} - 2 C_{HPO_4^{2-}} - C_{Cl^-} = 0$$

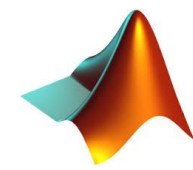
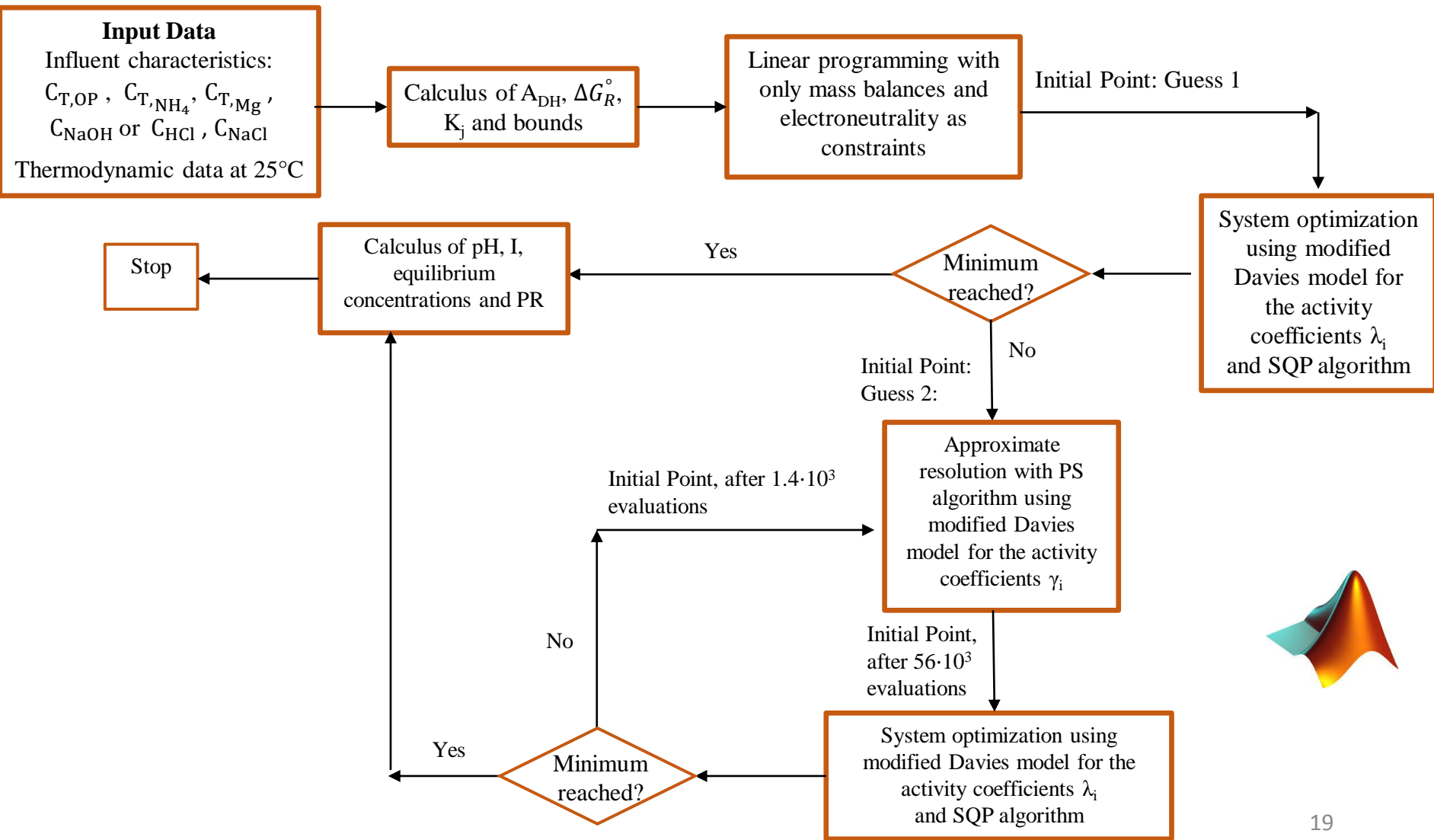
4 linear equality constraints in the proposed optimization problem

MASS AND ELECTRONEUTRALITY BALANCES

| Reaction j | Nonlinear equality constraints |
|---------------|--|
| (1) | $pK_1 + \log\left(\frac{\alpha_{H_2PO_4^-} \cdot \alpha_{H^+}}{\alpha_{H_3PO_4}}\right) = 0$ |
| (2) | $pK_2 + \log\left(\frac{\alpha_{HPO_4^{2-}} \cdot \alpha_{H^+}}{\alpha_{H_2PO_4^-}}\right) = 0$ |
| (3) | $pK_3 + \log\left(\frac{\alpha_{PO_4^{3-}} \cdot \alpha_{H^+}}{\alpha_{HPO_4^{2-}}}\right) = 0$ |
| (4) | $pK_4 + \log\left(\frac{\alpha_{H_2PO_4^-} \cdot \alpha_{Mg^{2+}}}{\alpha_{MgH_2PO_4^+}}\right) = 0$ |
| (5) | $pK_5 + \log\left(\frac{\alpha_{HPO_4^{2-}} \cdot \alpha_{Mg^{2+}}}{\alpha_{MgHPO_4}}\right) = 0$ |
| (6) | $pK_6 + \log\left(\frac{\alpha_{PO_4^{3-}} \cdot \alpha_{Mg^{2+}}}{\alpha_{MgPO_4^-}}\right) = 0$ |
| (7) | $pK_7 + \log\left(\frac{\alpha_{OH^-} \cdot \alpha_{Mg^{2+}}}{\alpha_{MgOH^+}}\right) = 0$ |
| (8) | $pK_8 + \log\left(\frac{\alpha_{NH_3} \cdot \alpha_{H^+}}{\alpha_{NH_4^+}}\right) = 0$ |
| (9) | $pK_9 + \log\left(\frac{\alpha_{OH^-} \cdot \alpha_{H^+}}{\alpha_{H_2O}}\right) = 0$ |
| (10) | $pK_{SP} + \log\left(a_{Mg^{2+}} \cdot a_{NH_4^+} \cdot a_{PO_4^{3-}}\right) = 0$ |

10 nonlinear equality constraints
in the proposed optimization
problem

SOLVING THE THERMODYNAMIC MODEL

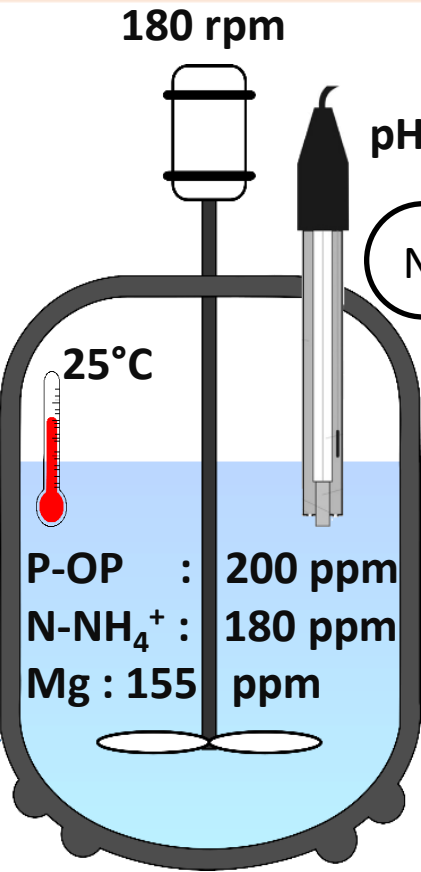


MODEL VALIDATION. EXPERIMENTAL PROCEDURE

Synthetic wastewater solution:
 $(\text{NH}_4)_2\text{HPO}_4$
 NH_4Cl

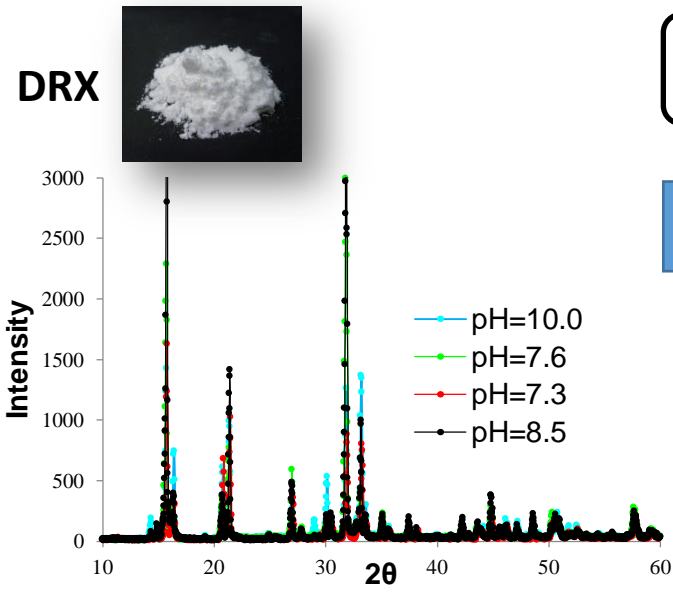


$\text{MgCl}_2 \cdot 6\text{H}_2\text{O}$

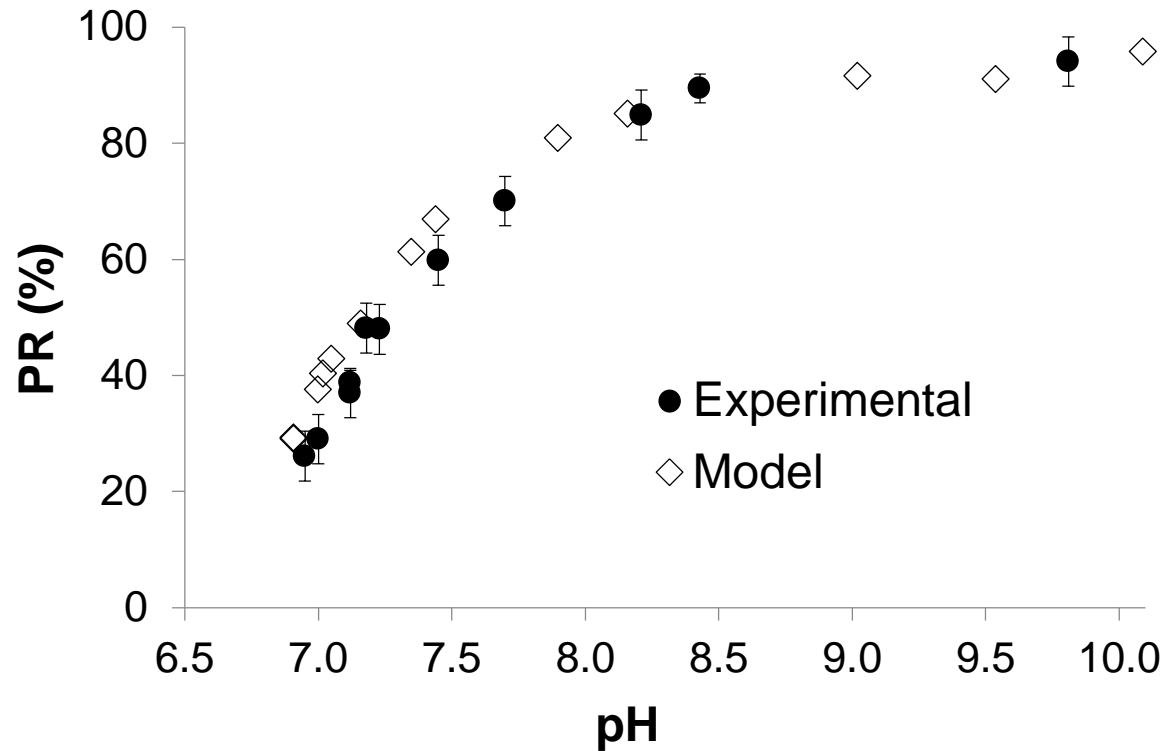


Reaction time:
 3 hours

Mg/P and N/P were 1.0 and 2.0

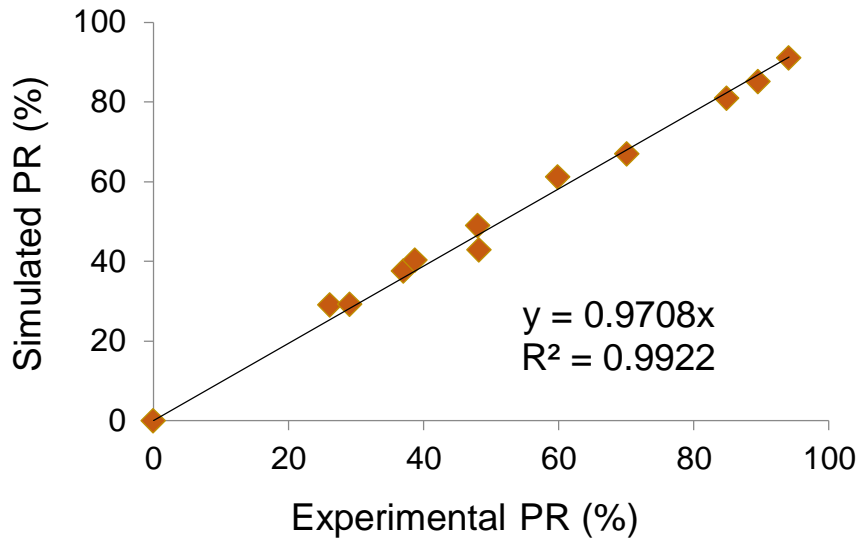


VERIFICATION OF THE PROPOSED MODEL



Predicting and measured PR (%) as struvite for **P-OP =200 ppm** at the molar ratios **Mg/P=1** and **N/P=2**, **T=25°C**.

VERIFICATION OF THE PROPOSED MODEL



Data exhibit a high correspondence between the experimental and simulated sets

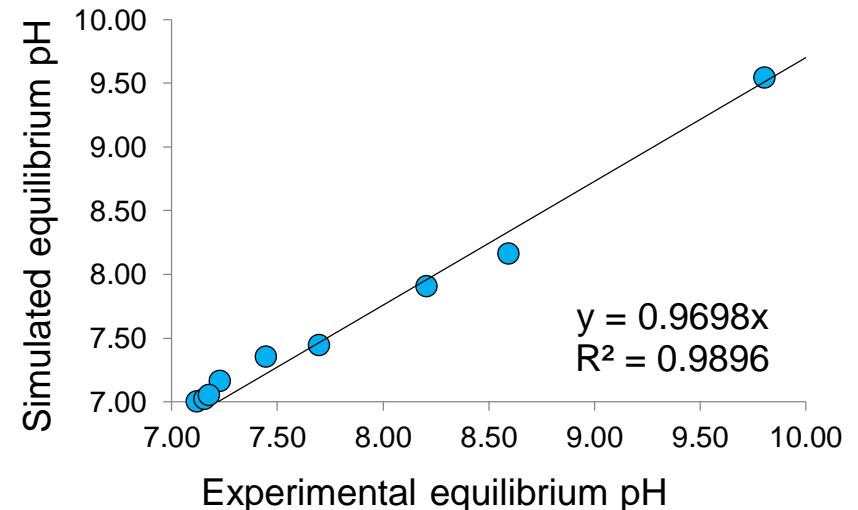
Linear correlations between pH obtained by the simulated data and experimental measurements.

Conditions:

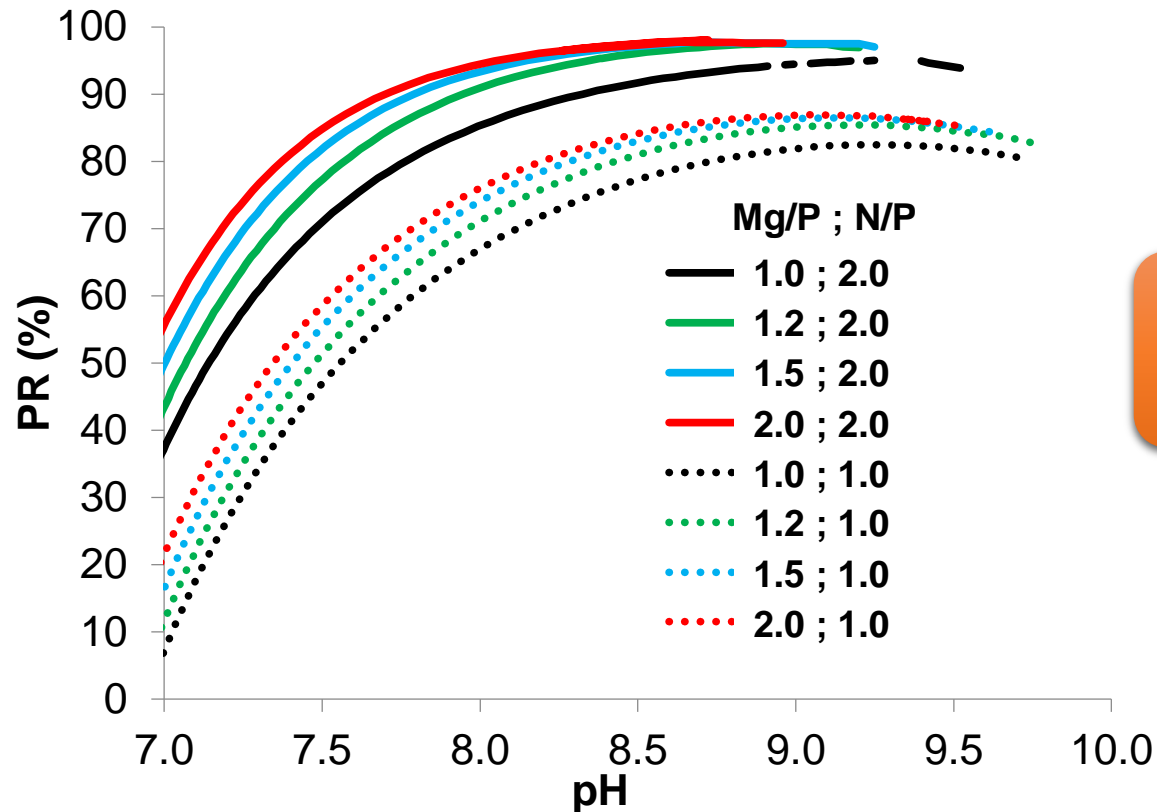
P-OP=200 ppm; Mg/P=1; N/P=2

T=25°C

Linear correlations between PR (%) obtained by simulated and experimental data.



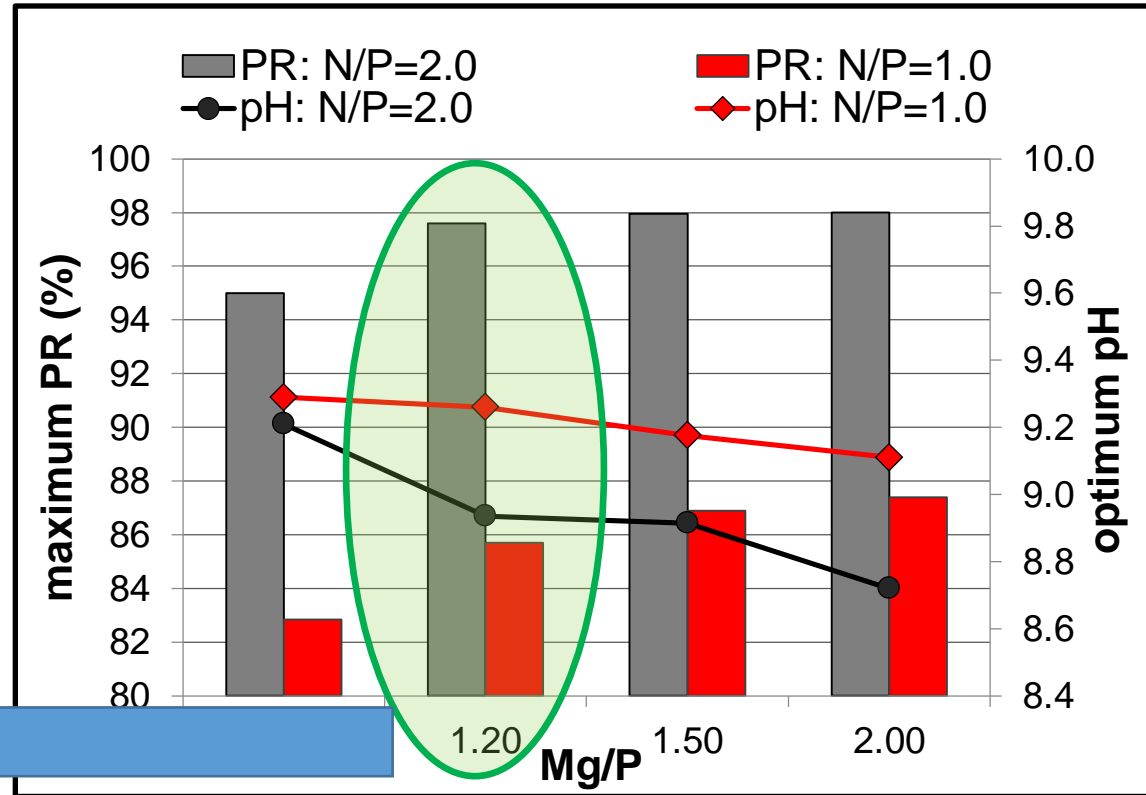
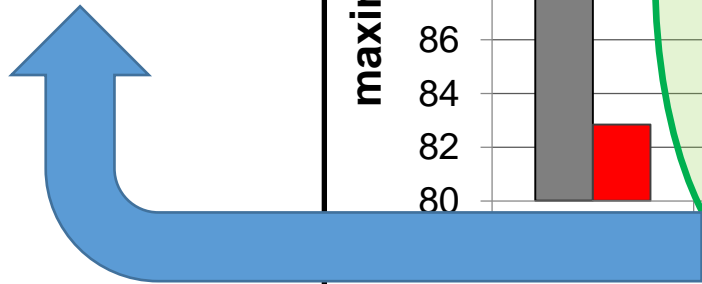
EFFECT OF MOLAR RATIO Mg/P AND N/P AT DIFFERENT pH ON THE PHOSPHORUS REMOVAL



Influence of molar ratios Mg/P (1.0, 1.2, 1.5 and 2.0) and N/P (1.0, 2.0) at different pH on the PR.

EFFECT OF MOLAR RATIO Mg/P AND N/P AT DIFFERENT pH ON THE PHOSPHORUS REMOVAL

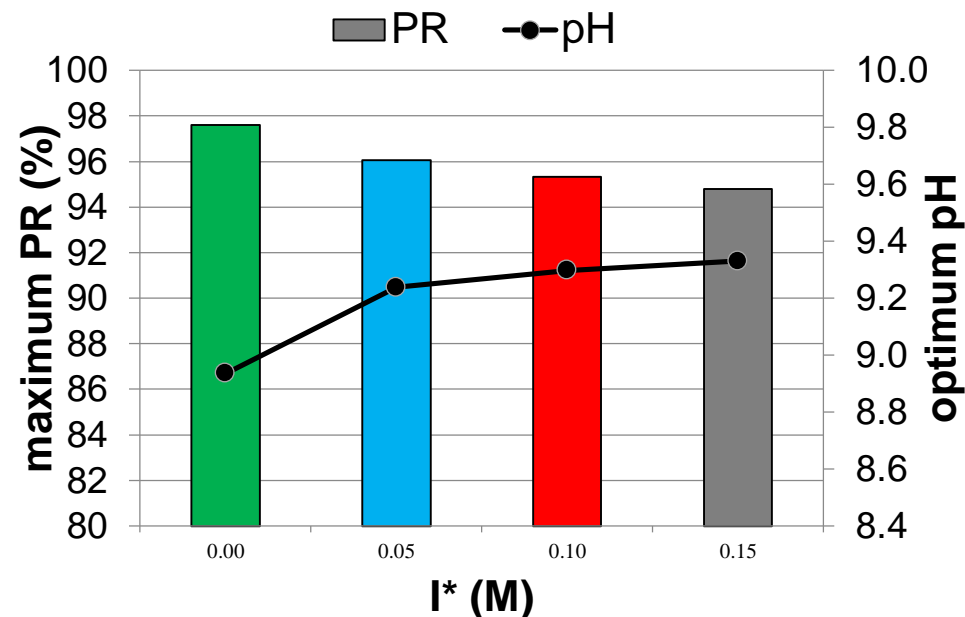
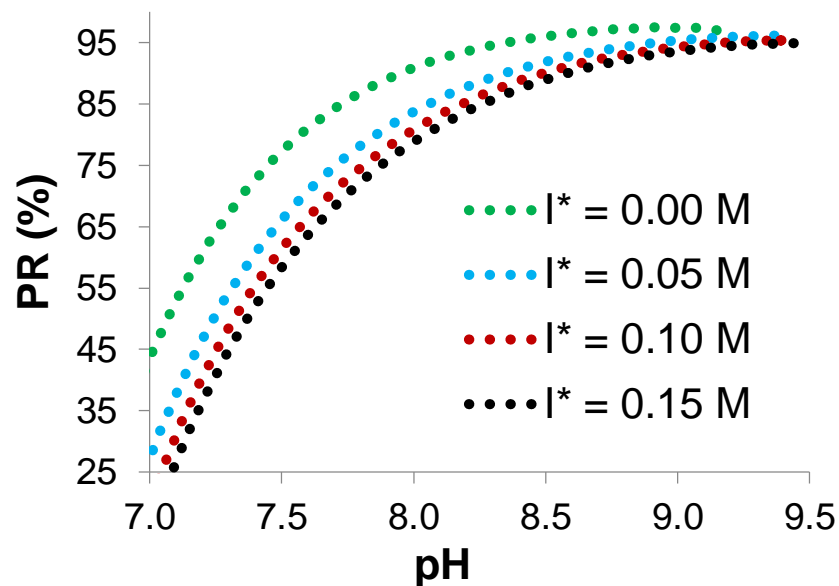
The higher relative improve $\Delta PR/(\Delta Mg/P)$ is obtained at molar ratio $Mg/P=1.20$ for both relations N/P evaluated.



P-OP=200 ppm
T=25°C

Optimal pH operating conditions and maximum PR (%) at molar ratio Mg/P (1.0, 1.2, 1.5 and 2.0) and N/P (1.0, 2.0).

EFFECT OF IONIC STRENGTH



Effect of ionic strength in the optimal pH operating conditions and maximum PR (%) at molar ratios $Mg/P = 1.2$, $N/P = 2$.

P-OP=200 ppm
T=25°C

CONCLUSIONS

1. A **hybrid optimization procedure** combining a **PS algorithm** and **SQP method** has been developed to **predict the potential P-OP removal as struvite** from a wastewater stream with **known chemical composition** and defined **operational precipitation conditions**.
2. The **predicted values matched fairly well with the experimental results** for **PR and equilibrium pH** (in range 7.0-10.0) and for the concentrations tested (P-OP: 200 ppm, Mg/P=1.0 and N/P=2.0).
3. For the **P-OP** concentration evaluated (**200 ppm**) the condition defined by molar ratios **N/P=2.0** and **Mg/P=1.20** result in a **good industrial operation candidate**: high PR (97.6%)) is achieved at a reasonably lower Mg concentration, which means lower reagents cost.
4. An **increment at ionic strength** **reduces the maximum PR** reached and increments the optimal pH.

Modelling the thermodynamic equilibrium of struvite precipitation using a hybrid optimization technique



PROGRAMA PARAGUAYO PARA EL DESARROLLO DE LA CIENCIA Y TECNOLOGÍA



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