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"Valuación de Inversiones en Infraestructura Eléctrica y Comportamiento Estratégico"

ANEXO 20
PGT 6.1 – Integración y Comparación de modelos de Opciones Reales (OR) y Teoría de Juegos (TJ), ABM y Dinámica de Sistemas (DS).

Agent-Based Modelling of Cost Efficient and Stable Transmission Grid Expansion Planning

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Abstract. Due to politically defined goals to raise the share of renewable energy, the landscape of electricity production has changed in recent years. Normally, a decision to invest in new generation capacity by generation companies is often based on profit maximization criteria. Criteria considering the costs resulting from the required expansion or construction of new transmission capacity are only playing a minor role, if any. This paper introduces an integrated model based on a multi-agent system to simulate the investment and decision behavior of the relevant entities in the liberalized energy market and their impact on social welfare. The interaction between the modelled market entities is based on a non-cooperative game theoretic approach. Its functionality is demonstrated within a small application example.

Keywords: Multi-agent systems · Macroeconomics · Simulation · Transmission grid · Expansion planning

1 Introduction

Investments in new production capacity in a liberalized energy market have a huge impact on the transmission and distribution grid, as well as on the price structure of the supplied electrical energy. Whereas generation companies (GenCos) are normally allowed to select their investment in new generation capacity and power plants based on their own business criteria, transmission system operators (TSOs) are regulated by governmental institutions. The fact, that GenCos are allowed to freely choose their investment point can lead to necessary cost-intensive transmission grid expansion and consequently to a decrease in social welfare.

Agent-based modelling (ABM) or Multi-Agent based simulation (MABS) has become popular in recent years to model and simulate the electricity market behavior. Its flexibility allows a wide range of application like analyzing generation expansion decisions in electricity markets by simulating the decentralized decision making process of GenCos [1] or to improve the planning of distribution grids [2, 3].

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Beside the use of MABS to improve planning of distribution grids, expansion planning of power grids in general has been and still is an important field of research. A good overview about the various economic and engineering issues and challenges on this field of research is given by Wu *et al.* in [4]. In the recent years, anticipative network planning models have been formulated to simulate the correlation between transmission grid expansion planning and investments in new generation capacity. While there is not yet many work related to this, already published work investigates the effects of transmission grid expansion strategies on investments of GenCos in new generation company.

Sauma and Oren [5, 6] propose and solve a three-stage approach for the investigation of the investment behavior of a transmission system operator (TSO) under the assumption that the TSO is anticipating the investment behavior of GenCos. Their proactive transmission expansion planning model includes the valuation of different grid expansion possibilities. In [7], Pozo *et al.* relate to this approach and continue its development. In their proposed approach, the transmission planner is able to consider the induced behavior by the GenCo investment and the electricity market equilibrium for new transmission grid expansion plans including demand uncertainty. A transmission grid expansion framework under consideration of the strategic behavior of GenCos is also proposed by Motamedi *et al.* in [8]. They propose an iterative, agent-based search algorithm to solve a multiple layer optimization problem. In [9], Yen *et al.* present a multi-agent based approach under the use of cooperative game theory and coalition forming. They investigate different outcomes of coalitions of different agents, which are responsible for the development of the transmission grid.

The approach presented in this paper differs from already existing work that a non-cooperative game is formulated as a framework for a MABS. The result is a MABS tool that is capable to simulate the strategic investment behavior of the electricity market participants while considering the regulation framework of the energy market. The goal is to find solutions for transmission grid planning considering the investment behavior of GenCos in a liberalized energy market. The main difference to existing work is that the market power of GenCos in a liberalized market with regulations is anticipated. To achieve this objective, the influence parameters of the individual market entities have been investigated and their objectives have been formulated. The modeled decision makers are interacting with each other in a predefined, non-cooperative game theoretic based simulation framework. The target of this approach is to propose a new simulation model to identify instruments and mechanisms, which are able to increase social welfare. One of the main objectives is to determine if there are any instruments in a regulated liberalized energy market to influence GenCos to invest in new generation capacity while reducing or avoid transmission capacity expansion. The combination of a MABS with the game theoretic framework creates a basic but highly expandable simulation tool.

The paper starts with a description of the assumptions and the theoretical concept of the model. Afterwards the developed MABS and its agents are introduced. To demonstrate its functionality, the results of a three node application case simulation are presented.

2 Assumptions and Theoretical Concept

The developed model assumes a general network topology with a DC power flow. Congestion or violation of the ($n-1$)-criterion on multiple lines are possible. The violation of the ($n-1$)-criterion is estimated as a branch usage equal or bigger than 70 % of its capacity [10]. It is assumed that all nodes are demand and generation nodes and that there is a constant load at every node over the whole simulation runtime. Furthermore, uncertainty is not included, which means that every market entity can exactly calculate the costs, revenue and any other parameters of an investment opportunity. For the sake of simplicity it is assumed that the electricity market is represented by an optimal power flow calculation.

In the simulation, the following sequence of events is assumed:

Step 1 – The regulator evaluates different regulation options for a specific period
Step 2 – The GenCo evaluates different investment projects and invests in the most profitable one while considering the resulting transmission grid expansion costs. The investment decreases its marginal cost of production.
Step 3 – If necessary the TSO invests in new transmission capacity.
Step 4 – Market operations are taking place.

The assumed sequence of events can be interpreted as a one-period investment cycle, whereas one period can be interpreted as one year. At the beginning of each year, the regulator evaluates its different regulation options for a specific period. Afterwards, every company evaluates the investments that are possible or have to be made during the year. If each company made their investment (or decided itself against to make an investment) the market operations are taking place until the end of the period. In the next period, the same sequence of events is taking place again while considering the results of the previous period.

The interactions between the different decision makers in electricity markets are modeled as a complete and perfect information leader-follower game. This includes that every player is able to observe the actions of the other players and to make its "rational" decision based on these decisions. The game consists of four players, representing the modeled market entities regulator, GenCo, TSO and a fictitious market operator (MO) which is responsible for market calculations. Due to the nature of non-cooperative game theory, every player has its own objectives and tries to maximize them. This intrinsic objective can be formulated in an objective function for each agent. To solve the game, a subgame perfect Nash equilibrium has to be found.

In the proposed approach, each round of the game represents one period (*e.g.* one year) and all actions are made sequentially. To simulate more than one period, the sequence of every player's action has to be repeated, considering the simulation results from the previous period.

Hence at the end of each year, respectively shortly before the end of each year, the regulator decides if it will change the regulations for the following year or not. After this step, the GenCo decides if it is going to invest in new generation capacity or not. Depending on the GenCo decision, the TSO has to invest in new transmission capacity to prevent congestion. It is assumed, that the TSO has to connect a new power plant of the GenCo and is also responsible for the stability of the transmission grid. Following the decision of the TSO, market operations are taking place. In this sequence of actions, every player tries to determine its best strategy anticipating the best strategy of the other players and vice versa. Thus, they choose a strategy that performs better against other strategies. The resulting strategy of each player is not necessarily a strategy that maximizes its payoff but a strategy that maximizes its payoff considering the reaction of the other players. If no player has an incentive anymore to pick another strategy, a subgame perfect Nash equilibrium is found. If such an equilibrium exists, it represents a specific strategy combination and determines the solution of the game.

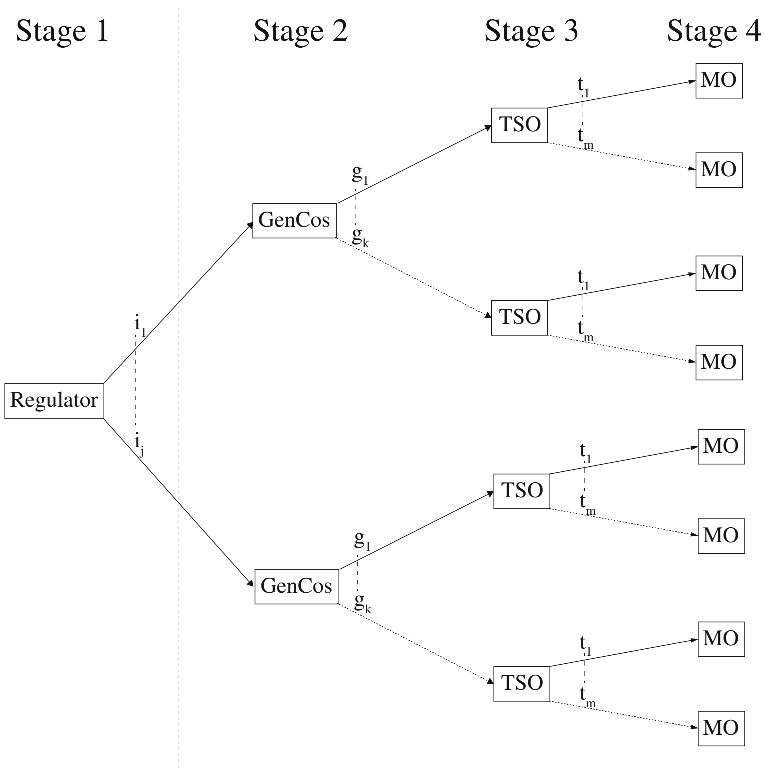

Fig. 1. Decision tree of the developed model

The tree in Fig. 1 corresponds to the previously introduced sequence of actions, it cannot be used to find the best solution for every player in the game. Hence, the concept of backward induction is used. This concept is based on the idea that one has to identify the “bottom-most” (here: stage 4) equilibria of the subgame trees and assume, that those equilibria will be played. In the next step, the equilibria of the subgames on the next higher stage (here: stage 3) have to be found. This procedure has to be done for every stage until the top of the tree is reached.

Figure 2 displays the schematic sequence of the backward induction solution process. As observable, the number of calculations and iterations to solve the game highly depends on the number of players and options of each player in the different stages. Additionally, the time for solving the game computationally depends on the complexity of the objective functions of each player on each stage, as well as the number of players on each stage and their number of options.

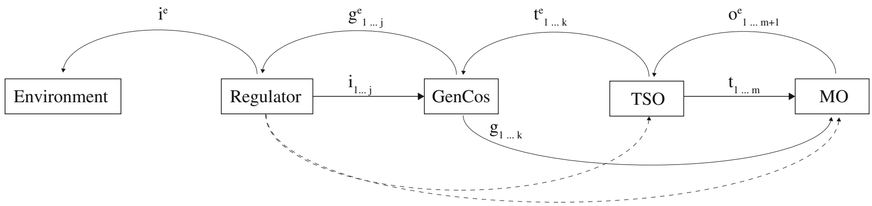

Fig. 2. Schematic sequence of the solution process via backward induction

For every of its *j*-options $\left(t_{1\ldots j}\right)$ the RegulatorAgent (stage one) informs the GenCoAgent(s) (stage two) about its regulation(s). Additionally, it is also possible but not obligatory to let the regulator inform the TSOAgent (stage three) as well as the MarketOperatorAgent (stage four) about their regulation(s). Then, the GenCoAgent informs the MarketOperator agent about its first of its *k*-options $\left(g_{1\ldots k}\right)$. The MarketOperatorAgent then reports its equilibrium option o_1^e based on its objective function to the TSOAgent. The TSOAgent then selects its first option t_1 and sends this option to the MarketOperatorAgent which reports its equilibrium option o_2^e based on g_1 and t_1. This process is repeated until t_m is reached. The TSOAgent then selects the best result of the m results and sends t_1^e to the GenCoAgent. Then, the GenCoAgent selects g_2 and the whole optimization process through stage four and three takes place again. This process goes on until g_k is reached. The GenCoAgent then reports its equilibrium g_1^e to the RegulatorAgent. The RegulatorAgent then selects i_2 and the whole optimization process through stages four, three and two are taking place again. This activity is done until the RegulatorAgent reaches i_j. The RegulatorAgent then selects i^e according to its objective function as the result of the optimization process for the specific period and sends it to the EnvironmentAgent. The selection of i^e determines the selected option in all subsequent stages. This is because every agent on the subsequent stages select its option that performs best as a response to the options selected by the other agents. Hence, at this point, a Nash equilibrium through backward induction for the simulated period is found. It is important to take into account, that all the optimization steps explained above have to be made for every period that is simulated. That means in effect, playing the game several times can simulate a multi-period-optimization problem.

3 Agents

The proposed MABS in the game framework consists of five agents, whereas one agent, the EnvironmentAgent, can be considered as the “game master” (a non-player agent) and four player agents. It has been developed with the use of the Java Agent Development Framework (JADE) [11].

3.1 EnvironmentAgent

The EnvironmentAgent is necessary to ensure a stable synchronization between the player agents by providing them with all necessary information about the grid, generators, investment possibilities and investment costs. Furthermore, it ensures an orderly sequence of the simulation by sending a start signal to the player agents and settling each period and its specific simulation results. If required, it is able to provide the user with a GUI of the investigated grid.

3.2 MarketOperatorAgent

The MarketOperatorAgent simulates the market behavior. Its goal is to solve a DC optimal power flow calculation (DC OPF). Whereas this means that it has no objective function in the classical sense, the calculation of the DC OPF and the provision of the resulting data can be considered as its goal function. The optimal power flow calculation is based on work by Sun and Tesfatsion [12, 13]. The MarketOperatorAgent receives its initial data at the beginning of each period from the EnvironmentAgent. During the simulation, it receives its data from the GenCoAgent and the TSOAgent. The optimal generation dispatch, the prices and the branch usage are then calculated through the DC OPF calculation. The resulting data, like generator and branch usage and production costs, is assumed to represent the optimal market behavior. It is send to the TSOAgent for further calculation. Hence, the main objective of the MarketOperatorAgent can be described as “the provision of the result of an optimal market behavior”.

3.3 TSOAgent

The TSOAgents objective is to minimize its overall costs while supplying power over the transmission grid to satisfy demand. The costs incurred resulting from investments to secure system stability and transmission grid expansion due to congestion or violation of the (n-1)-criterion. The high complexity of the transmission grid expansion cost calculation in reality requires a few assumptions and simplifications to model the behavior of a TSO. It is assumed that the TSO is obligated to provide a secure network. It does not consider any costs for maintenance of already existing transmission lines. Only investments in new transmission lines or upgrades of existing transmission lines are considered. Furthermore, it is assumed, that the TSOs investment options are limited to one possible investment material and that every investment and its costs take place in the contemplated period. Under these assumptions, the TSOAgent investigates the resulting grid data it receives from the MO. If there is any congestion or violation of the (n-1)-criterion it executes a grid optimization method to remedy the problem in the branch configuration.

3.4 GenCoAgent

The GenCoAgents objective is to maximize its revenue from the production and sale of electricity of a new power plant. For the sake of simplicity, it is assumed that already existing power plants that are owned by a specific GenCo are not taken into account. Hence, the GenCoAgent just considers the possible investment projects and their expected revenue.

To do so, the GenCoAgent solves the following objective function (The formula is based on [15].):

$$\max(\mathrm{Rev} - \mathrm{IC}(\mathrm{PC}) - \mathrm{varCo}) \tag{1}$$

Whereby Rev is the expected revenue of an investment project, IC(PC) are the investment costs which depend on the production capacity of the new power plant and possible costs due to necessary transmission grid capacity expansion if given. VarCo are the expected variable costs during the expected lifetime of the power plant. The expected revenue is calculated using the expected utilization rate and the expected energy price provided by the DC OPF of the MarketOperatorAgent.

The GenCoAgent calculates the internal rate of return (IRR) to compare the different investment possibilities in power plants. It then selects the investment possibility with the highest IRR and compares the value with an individual, user-defined minimum accepted rate of return (MARR). If the IRR is higher or equal to the MARR, it selects the specific investment. If the IRR is lower than the MARR, the GenCoAgent will not invest in one of the available investment possibilities.

For the calculation of the generation costs during the optimal power flow calculation, the following generator total cost function for a generator i, presented in [14], is used.

$$\mathrm{TC}_i(\mathrm{p}_{\mathrm{G}i}) = \mathrm{a}_i * \mathrm{p}_{\mathrm{G}i} + \mathrm{b}_i * \mathrm{p}_{\mathrm{G}i}^2 + \mathrm{FCost}_i \tag{2}$$

Here $\mathrm{p}_{\mathrm{G}i}$ denotes real power produced by generator i, a_i denotes costs that are proportional to the generated power, b_i denotes a cost depending efficiency factor and FCost_i denotes fix costs that are independently from the power production.

RegulatorAgent. The RegulatorAgent is a player agent representing the behavior of the regulator in the real world. Furthermore, it is the leader for all players in the game. Its main objective is to increase social welfare. Social welfare in the context of the developed model is defined under the assumption, that there is a constant demand on every node of the grid independent of price changes. Hence, the objective function of the RegulatorAgent can be expressed as follows:[1]

$$\min\left(\sum\nolimits_{i\in G} C_i(p_{Gi}) + \sum\nolimits_{k\in TL_n} C_k\right) \tag{3}$$

Whereby $\sum_{i\in G} C_i(p_{Gi})$ is the sum of all electricity generation variable costs to satisfy the demand on every node and $\sum_{k\in TL_n} C_k$ is the sum of all costs for new transmission lines which are necessary due to investments in new power plants by the GenCo.

To fulfill its objective, the RegulatorAgent is interested in influencing the market participants to increase social welfare. In the developed approach, it has the ability to influence the GenCoAgent's investments by setting up a specific splitting ratio. This splitting ratio divides the costs for new transmission capacity investments between the GenCo and the TSO (respectively the consumer). In the base case, all costs for extending the transmission grid are worn by the TSO (or the consumer due to higher electricity prices). The RegulatorAgent then changes the ratio by splitting up the costs of transmission grid extension, whereby 10% of a new line due to an investment in a new power plant have to be paid by the GenCo and 90% have to be paid by the TSO (or the consumer). This process goes on until the RegulatorAgent found a splitting ratio, which leads to an equal or increased social welfare compared to the status quo.

4 Application Example

In the following an application example to demonstrate the functionality of the developed model is presented. The subject of investigation is a liberalized energy market that consists of a three-node grid. The existing configuration of the grid, the technical details of the branches, loads and power plants are presented in the following tables. Only one period is simulated and it is assumed, that every investment in transmission lines and generation capacity takes place at the beginning of the period. Furthermore, a constant demand at every node over the whole simulation period is assumed (Tables 1, 2, and 3).

Table 1. Sample case branch configuration

Parameter	Element		
	Branch 1	Branch 2	Branch 3
Start node	1	1	2
End node	2	3	3
Length [km]	20	20	20
Reactance	0,2	0,2	0,2
Capacity [MW]	400	450	600

[1] The formula is based on [16].

Table 2. Sample case load configuration

Parameter	Element		
	Load 1	Load 2	Load 3
ID	L1	L2	L3
Node	1	2	3
Load [MW]	300	500	1000

Table 3. Sample case generator configuration

Parameter	Element		
	Generator 1	Generator 2	Generator 3
ID	G1	G2	G3
Node	1	2	3
Fix costs [€/h]	0	0	0
Cost coefficient a [€/MWh]	13	60	40
Cost coefficient b [€/MW^{2}h]	0.00001	0.00001	0.00001
Minimum capacity [MW]	0	0	0
Maximum capacity [MW]	770	1300	500
Initial production [MW]	0[a]	0[a]	0[a]

[a]Set by the MarketOperatorAgent as part of the model initialization

After an initial DC OPF, the data presented above leads to the following graphical representation in the GUI of the EnvironmentAgent (Fig. 3).

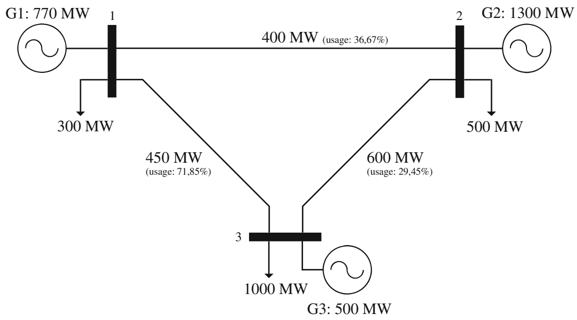

Fig. 3. Initial grid configuration of the sample case

Furthermore, it is assumed that the GenCo considers the following three investment options in new generation capacity whereby every investment option represents a new power plant. Furthermore, the GenCo assumes different parameters for each investment option (Table 4).

With the parameters for the different investment options given in Table 4, the GenCoAgent calculates the revenue and the rate of return of every investment. For the

Table 4. Sample case GenCo investment options

Parameter	Option		
	Investment Option 1	Investment Option 2	Investment Option 3
ID	1	2	3
Node	0	0	0
Minimum capacity [MW]	150	100	200
Maximum capacity [MW]	0	0	0
Assumed freight costs	0	0	0
Assumed Fix costs [€/MW]	5.190.000	4.000.000	4.000.000
Assumed invest costs [€/MW]	20	13,5	12,5
Assumed runtime [years]	1	2	3

sample case the best rate of return is given by the investment option 1 at node 1. Hence, without any regulations the GenCo tends to invest in 150 MW generation capacity at node 1.

In the following, only the options and the corresponding results of the RegulatorAgent calculations are displayed. This is because in its function as the game leader, its decision is crucial for the decisions of all subsequent game stages. In this application case, the RegulatorAgent has 11 different regulation options. Every option distributes the costs of new transmission capacity between the TSO (tc_i) and the GenCo (gc_i) whose investment in generation capacity requires the grid capacity expansion.

Table 5. Optimization results for the eleven regulator options of the application case

Option configuration	Regulator option (gc_i, tc_i)	Overall system costs [Euro/period][a]
Regulator option 1–11	(0,0; 1,0)	$4{,}7079*10^8$
GenCo option*	(0,1; 0,9)	$4{,}7079*10^8$
TSO option*	(0,2; 0,8)	$4{,}7079*10^8$
Market option*	(0,3; 0,7)	$4{,}7079*10^8$
	(0,4; 0,6)	$4{,}7079*10^8$
	(0,5; 0,5)	$4{,}7079*10^8$
	(0,6; 0,4)	$4{,}7079*10^8$
	(0,7; 0,3)	$4{,}7079*10^8$
	(0,8; 0,2)	$4.36509*10^8$
	(0,9; 0,1)	$4.36509*10^8$
	(1,0; 0,0)	$4.36509*10^8$

[a]All values rounded.

Table 5 displays the different results the RegulatorAgent receives from its subsequent stages. For the first eight options the overall system costs are constant, while for the last three options the value is quite smaller which implies a higher level of social welfare. According to its goal to increase social welfare (respectively minimize the overall system costs) while regulating as little as possible the RegulatorAgent selects the option with the splitting ratio (0,8; 0,2). This means in effect, that in the case in which the GenCo has to pay 80 % new transmission line costs due to its power plant investment at node 1, it tends to invest in another location and tries to avoid an investment that would need transmission grid expansion.

Since the RegulatorAgent made its decision, a result for the simulated sample case has been found. Now, every agent chooses its best option according to its objective function while anticipating the decision of the other agents. These decisions lead to the following changed graphical representation of the grid configuration in the EnvironmentAgent GUI which represents the static operation point after the DC OPF.

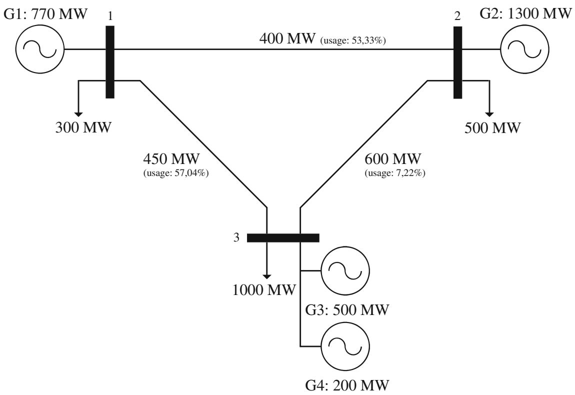

Fig. 4. Graphical grid representation of the final optimal grid of the sample case

There are two important things that have to be noted from the final result of the simulated period represented by Fig. 4. Firstly, in contrast to its best investment option, which is 150 MW at node 1, the final investment in new generation capacity of the GenCo is 200 MW at node 2. That is because at the point, where the GenCo is forced to pay 80 % of the transmission capacity extension costs, the break-even point for the investment at node 1 is exceeded. Secondly it is observable that no investment in new transmission capacity by the TSO has been made. While any other investment in new generation capacity would lead to necessary investments in transmission capacity by the TSO, the investment at node 2 reduces the power flow on branch 2. Due to this reduction of the branch usage an investment in new transmission capacity is not necessary anymore.

5 Analysis and Outlook

The application example above shows that depending on the grid configuration and transmission expansion planning scenarios a regulation of investments in new generation capacity can increase social welfare. Such a growth in social welfare depends mainly on the power that is granted to the regulator. If the regulator is restricted to the responsibility to guarantee free markets and if it has no instruments to regulate the market behavior of market entities directly, an increase is not taking place inevitably.

But if the regulator has the power to restrict the investment behavior of GenCos, social welfare could be increased. In the very simple application example, this fact is shown by giving the regulator the power to set up a "splitting ratio" which distributes the share of costs of transmission grid expansion to the transmission grid operator or the consumer and the investing generation company. In further works, the developed approach needs to be extended in different directions to consider the complex interdependencies of the electricity market and grid. First of all, the modeling of the market has to be extended to provide a realistic behavior of the power plant production including a model for an energy exchange. This also includes the consideration of a more realistic simulation of the grid by using time-series base power flow calculations considering a more fluctuating load and demand. Furthermore, a possible regulatory framework as well as different mechanisms to influence GenCos and TSOs has to be developed. In a second step, the evaluated grid has to be justified and extended to represent a more realistic grid. Consequently, the objectives of the agents have to be justified to consider the complex dependencies of a more challenging test grid.

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