2018 International Conference on Unmanned Aircraft Systems (ICUAS)
Dallas, TX, USA, June 12-15, 2018

Attitude Tracking of a Tri-Rotor UAV based on Robust Sliding Mode with Time Delay Estimation

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Abstract— **The paper presents a robust sliding mode with time delay estimation method for controlling the attitude of a tri-rotor unmanned aerial vehicle (UAV) in presence of uncertainties and disturbances. The proposed control algorithm allows high accuracy tracking since a good disturbance estimation is provided using time delay estimation method and allows chattering reduction. The stability analysis of the closed-loop system is presented using the theory of Lyapunov. Finally, two numerical simulations are presented in the presence of disturbances to show the effectiveness of the proposed nonlinear control scheme.**

I. Introduction

In the last decades, the interest in the development of unmanned aerial vehicles (UAV) and the implementation of modern flight control systems have been justified mainly by a wide range of applications, both military and civil [1]. One of the civil applications is the aerial photogrammetry for the topographic study mainly in planialtimetric surveys and volume calculation, justified mainly by being a technique of fast deployment, low implementation cost and accuracy comparable to traditional aerial photogrammetry techniques based on light detection and ranging (LIDAR), being the main limitation of this latter technique its high cost [2], [3].

In aerial photogrammetry applications where a good coverage of topographic analysis is required, a factor to be taken into account is the flight autonomy of the UAV, being the fixed wing ones probably the best option in terms of flight autonomy. However, in many cases it is necessary to be able to take off and land in small areas with air obstructions, being the multi-rotors an interesting alternative because they have the capability of vertical take-off and landing (VTOL) [4]. In this context, the proposed scheme is based on a UAV with three rotors like the one shown in Fig. 1.

From the point of view of control, different techniques have been tackled to date from linear proportional-integral-derivative and linear-quadratic controllers [5], [6], to modern nonlinear control techniques based on neural networks, fuzzy control, state-based observer control and more recently, sliding mode control (SMC). A taxonomy of modern control techniques implemented on UAVs can be found in [7]. The main contribution of this paper is the theoretical study of new nonlinear control strategies based on SMC approach [8]. SMC is a finite-time controller known for his robustness against uncertainties and disturbances and for his simplicity of derivation. The basic idea of this variable structure control (VSC) is to force the system trajectories to reach a user-defined sliding surface and to remain on it until converging to the equilibrium point. However, the real time implementation of this control technique is limited by its major drawback, the well-known chattering phenomenon [9], [10]. In order to solve this problem, many developments have been published, we cite in this context:

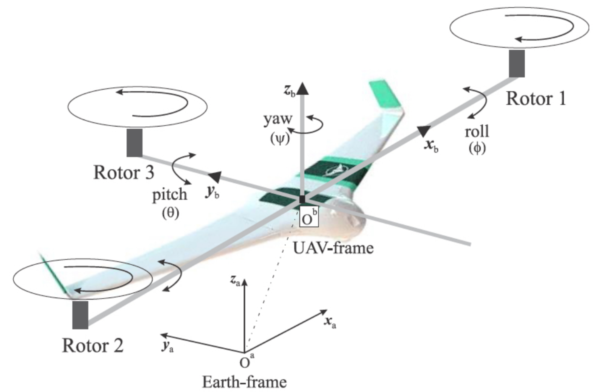

Fig. 1. Tri-rotor UAV.

- Sliding mode based on a boundary layer [11]. The idea consists on using continuous functions such as saturation or hyperbolic tangent instead the signum function. This method reduces the chattering phenomenon, but the finite-time convergence property is lost in this case which is very desirable while controlling fast nonlinear systems.
- Observer-based sliding mode control [12]. This method reduces the problem of designing a robust controller into the problem of designing a robust observer. It means that if the uncertainties estimation is not exact, the desired performances will be affected.
- Higher order sliding mode (HOSM) [13]. In case of second order systems as UAVs, we talk about the second order sliding mode (SOSM) [14], [15]. The basic idea is to make the signum function acting on the first time derivative of the control input, then, by integrating the control input becomes continuous. This approach allows

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chattering reduction and higher precision. However, the required informations are increased which make the implementation difficult.

Recently, Kali et al., propose a combination of SMC with time delay estimation (TDE) method [16]–[19]. The main advantage of the proposed controller is the chattering reduction. This method allows the choice of small values of the switching gains since the uncertainties and disturbances are well estimated in a simple way using TDE. In this paper, this control algorithm will be derived for the high accuracy attitude tracking and stabilization problem of a tri-rotor UAV system in the presence of disturbances and unknown dynamics.

The rest of the paper is organized in four sections. The considered tri-rotor UAV is described and his mathematical attitude model is given in section II. In section III, the combination of sliding mode control with time delay estimation method is designed and the stability analysis is carried out. In section IV, numerical simulations are presented for attitude stabilization and tracking in the presence of disturbances to prove the effectiveness of the proposed method. Finally, the conclusion is drawn in the fifth section.

II. Tri-rotor attitude model

The main advantage of the tri-rotor UAVs is that they require less motors than the other proposed UAV systems such as four-rotor or six-rotor drones. This advantage allows reduction in volume, weight and energy consumption. The two rotors placed in the forward part of the tri-rotor rotate in opposite direction with respect to the third rotor placed in the backward part.

The attitude model of the considered tri-rotor can be expressed by the following equation:

$$J\,W\,\ddot{\Theta}(t) + J\,\dot{W}\,\dot{\Theta}(t) + \Big(W\,\dot{\Theta}(t) \times J\,W\,\dot{\Theta}(t)\Big) = \tau(t) \quad (1)$$

where $\Theta(t) = [\phi(t), \theta(t), \psi(t)]^T$ denotes the Euler angles (roll $\phi(t)$, pitch $\theta(t)$ and yaw $\psi(t)$), $\tau(t) = [\tau_\phi(t), \tau_\theta(t), \tau_\psi(t)]^T$ represents the roll, pitch and yaw torques, $J = \text{diag}(I_x, I_y, I_z)$ is the diagonal inertia matrix while W is the Euler matrix and $\dot{W}$ represents its first time derivative. The matrix W is defined by:

$$W = \begin{bmatrix} 1 & 0 & -\sin(\theta(t)) \\ 0 & \cos(\phi(t)) & \cos(\theta(t))\sin(\phi(t)) \\ 0 & -\sin(\phi(t)) & \cos(\theta(t))\cos(\phi(t)) \end{bmatrix} \quad (2)$$

Moreover, the control torque inputs can be expressed as follows:

$$\begin{aligned} \tau_\phi(t) &= l_2\,(f_1 - f_2) && (3) \\ \tau_\theta(t) &= -l_1\,(f_1 + f_2) + l_3\,f_3\cos(\alpha) && (4) \\ \tau_\psi(t) &= -l_3\,f_3\sin(\alpha) && (5) \end{aligned}$$

where α represents the tilting angle of the third rotor placed in the backward part, f_i for $i = 1, 2, 3$ is the thrust generated by the rotor i, l_i for $i = 1, 2, 3$ are given in Fig. 2.

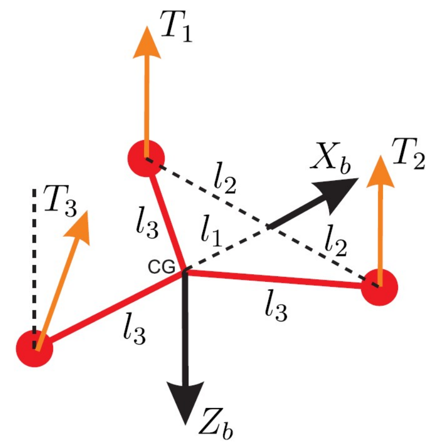

Fig. 2. Reference system for the tri-rotor UAV.

The control torque inputs given by (3), (4) and (5) can be written in a matrix form as follows:

$$\begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} l_2 & -l_2 & 0 \\ -l_1 & -l_1 & l_3\cos(\alpha) \\ 0 & 0 & -l_3\sin(\alpha) \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \quad (6)$$

Hence, each force is obtained using the inverse of the above equation as:

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \frac{-1}{l_2} & \frac{1}{l_1} & \frac{\text{ctg}(\alpha)}{l_1} \\ \frac{1}{l_2} & \frac{1}{l_1} & \frac{\text{ctg}(\alpha)}{l_1} \\ 0 & 0 & \frac{2}{l_3\sin(\alpha)} \end{bmatrix} \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} \quad (7)$$

III. Controller design

In this section, the proposed controller for the attitude tracking of the tri-rotor UAV based on a robust SMC with TDE method will be designed. The control objective is to ensure that the attitude positions defined by Euler angles $\phi(t), \theta(t), \psi(t)$ track with high precision even in presence of uncertainties and disturbances the bounded desired attitude trajectories $\phi_d(t), \theta_d(t), \psi_d(t)$. The architecture of the closed-loop system is represented in Fig. 3.

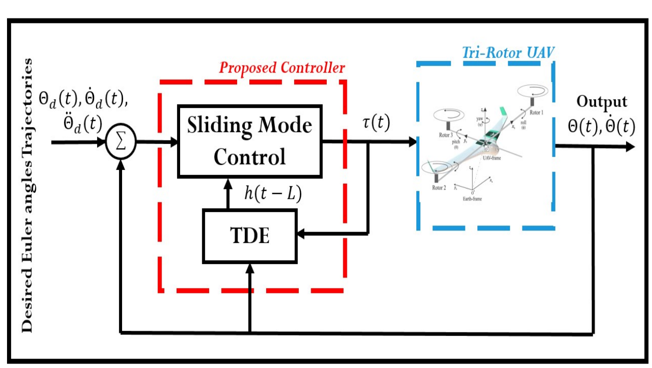

Fig. 3. Block diagram of the closed-loop tri-rotor UAV.

For simplicity, let us introduce $x(t) = [x_1^T(t), x_2^T(t)]^T$ as attitude state variables with $x_1(t) = [\phi(t), \theta(t), \psi(t)]^T$ and $x_2(t) = [\dot{\phi}(t), \dot{\theta}(t), \dot{\psi}(t)]^T$. Then, the equation of motion is given by:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= f(x(t)) + g(x(t))u(t) + h(t) \end{aligned} \tag{8}$$

Comparing the above equation with (1) gives the following equivalences:

$$f(x(t)) = -(JW)^{-1}\left(J\dot{W}\dot{\Theta}(t) + \left(W\dot{\Theta}(t) \times JW\dot{\Theta}(t)\right)\right)$$

$$g(x(t)) = (JW)^{-1},\ u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} = \begin{bmatrix} \tau_\phi(t) \\ \tau_\theta(t) \\ \tau_\psi(t) \end{bmatrix}$$

and $h(t) \in R^3$ denotes the uncertain vector caused by the wind disturbances, unmodelled dynamics.

In the following, the control law that will force the system trajectories $x_1(t)$ to converge to the the desired trajectories $x_{1d}(t) = [\phi_d(t), \theta_d(t), \psi_d(t)]^T$ is designed based on the following assumptions:

- **Assumption 1**: The attitude position trajectories $x_1(t)$ and their first time derivative $x_2(t)$ are available for measurements.
- **Assumption 2**: The desired trajectories $x_{1d}(t)$ and their first and second time derivatives $x_{2d}(t), \dot{x}_{2d}(t)$ are limited.
- **Assumption 3**: The uncertain functions $h_i(t)$ for $i = 1, 2, 3$ are globally Lipschitz:

$$\|\dot{h}_i(t)\| \leq \Delta H_i$$

The proposed controller is designed such in [16], [17]. Its goal is to force the system trajectories to converge to the user-chosen sliding surface defined by:

$$\begin{aligned} S(t) &= \dot{e}(t) + \lambda\, e(t) \\ &= x_2(t) - x_{2d}(t) + \lambda\,(x_1(t) - x_{1d}(t)) \end{aligned} \tag{9}$$

where $\lambda = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)$ is a diagonal positive definite matrix.

$$\begin{aligned} \dot{S}(t) &= \ddot{e}(t) + \lambda\dot{e}(t) \\ &= \dot{x}_2(t) - \dot{x}_{2d}(t) + \lambda\,\dot{e}(t) \\ &= f(x(t)) + g(x(t))\,u(t) + h(t) - \dot{x}_{2d}(t) + \lambda\,\dot{e}(t) \end{aligned} \tag{10}$$

The proposed robust SMC with TDE is obtained by resolving $\dot{S}(t) = -K\,\mathrm{sign}(S(t))$ and by substituting the uncertain vector $h(t)$ by its estimates obtained using TDE method as follows:

$$u(t) = u_{eq}(t) + u_{sw}(t) \tag{11}$$

with:

$$u_{eq}(t) = g(x(t))^{-1}\left[\dot{x}_{2d}(t) - \lambda\dot{e}(t) - f(x(t)) - \hat{h}(t)\right] \tag{12}$$

$$u_{sw}(t) = -g(x(t))^{-1}K\mathrm{sign}(S(t)) \tag{13}$$

where $K = \mathrm{diag}(K_1, K_2, K_3)$ is a diagonal positive definite matrix, the signum function $\mathrm{sign}(S(t))$ is defined by:

$$\mathrm{sign}(S(t)) = \begin{cases} 1, & \text{if } S(t) > 0 \\ 0, & \text{if } S(t) = 0 \\ -1, & \text{if } S(t) < 0 \end{cases} \tag{14}$$

and $\hat{h}(t)$ is the approximation of $h(t)$ obtained using TDE method:

$$\begin{aligned} \hat{h}(t) &\cong h(t-L) \\ &= \dot{x}_2(t-L) - f(x(t-L)) \\ &\quad -g(x(t-L))\,u(t-L) \end{aligned} \tag{15}$$

where L is the delay. We can notice that the precision of the estimation becomes higher with small value of L. The smallest value that can be chosen for the delay in real time implementation is the sampling time.

Theorem 3.1: If the switching gains are chosen such as the following condition is verified:

$$K_i > L\,\Delta H_i \text{ for } i = 1, 2, 3 \tag{16}$$

Then, the proposed robust controller given in (11) ensures the convergence of the sliding surface in a finite-time smaller than:

$$T_{c,i} = \frac{|S_i(0)|}{K_i - \delta_i} \text{ for } i = 1, 2, 3. \tag{17}$$

Proof: The closed loop error dynamics is obtained by replacing (11) in the tri-rotor model given in (8) as:

$$\dot{S}(t) = -K\,\mathrm{sign}(S(t)) + \varepsilon(t) \tag{18}$$

where $\varepsilon(t) = h(t) - \hat{h}(t)$ denotes the TDE error. Now, consider the following positive definite Lyapunov function:

$$V(t) = 0.5\,S^T(t)\,S(t) \tag{19}$$

Hence, its first time derivative is computed as follows:

$$\begin{aligned} \dot{V}(t) &= S^T(t)\,\dot{S}(t) \\ &= S^T(t)\,[\varepsilon(t) - K\,\mathrm{sign}(S(t))] \\ &= S^T(t)\left(L\,\dot{h}(t) - K\,\mathrm{sign}(S(t))\right) \\ &= \sum_{i=1}^{3} L\,S_i(t)\,\dot{h}_i(t) - K_i|S_i(t)| \end{aligned} \tag{20}$$

Based on **Assumption 3**, the following inequality can be set:

$$\begin{aligned} \dot{V}(t) &\leq \sum_{i=1}^{3}|S_i(t)|\left(L\,|\dot{h}_i(t)| - K_i\right) \\ &\leq \sum_{i=1}^{3}|S_i(t)|\,(L\,\Delta H_i - K_i) \end{aligned} \tag{21}$$

Finally, using the gains in (16), the above derivative of the Lyapunov function is negative definite.

To prove the finite time convergence, let us recall that:

$$S_i(t)\,\dot{S}_i(t) \leq |S_i(t)|\,(L\,\Delta H_i - K_i) \tag{22}$$

Then, dividing both sides of the above equation by $|S_i(t)|$ and integrating them between 0 and $T_{c,i}$ leads to:

$$\int_0^{T_{c,i}} \dot{S}_i(t)\mathrm{sign}(S_i(t))dt \leq \int_0^{T_{c,i}} (L\Delta H_i - K_i)\,dt \tag{23}$$

$$|S_i(T_{c,i})| - |S_i(0)| \leq (L\Delta H_i - K_i)\,T_{c,i} \tag{24}$$

Since $T_{c,i}$ is the required time to hit the sliding surface, which means that $S_i(T_{c,i}) = 0$. Then, the maximal convergence time is found as in (17). Here finishes the proof.

IV. Simulation results

In this section, simulation results are presented in order to demonstrate the effectiveness of the proposed controller based on SMC with TDE. We simulated on the attitude model (8) of the tri-rotor UAV. described in Section II using Matlab/Simulink software. The physical parameters of the used tri-rotor are given in Table I.

TABLE I
Physical parameters of the tri-rotor UAV

Parameters	Value
Mass moment of inertia in the x-axis,	$I_x = 0.111132$ Kgm^2
Mass moment of inertia in the y-axis,	$I_y = 0.13282$ Kgm^2
Mass moment of inertia in the z-axis,	$I_z = 0.249039$ Kgm^2
Length, l_1	0.275 m
Length, l_2	0.42 m
Length, l_3	0.52 m

In this part two numerical simulation has been performed. The first simulation involves the tri-rotor attitude stabilization from given initial angles. The initial Euler angles are chosen as:

$$\phi(0) = -0.4363 \, \text{rad}$$

$$\theta(0) = 0.3142 \, \text{rad}$$

$$\psi(0) = 0.3491 \, \text{rad}$$

In the second simulation, an attitude tracking is performed for the following desired angle trajectories:

$$\phi_d(t) = 0.17 \sin(\pi t) \, \text{rad}$$

$$\theta_d(t) = -0.17 \sin(\pi t) \, \text{rad}$$

$$\psi_d(t) = 0.52 \sin(\pi t) \, \text{rad}$$

For this two simulations, the chosen controller gains are:

$$\lambda = \text{diag}(10, 10, 10)$$

$$L = T_s = 0.003 \, \text{s}$$

$$K = \text{diag}(2, 2, 2)$$

Moreover, the disturbance considered on each Euler angle is depicted in Fig. 4.

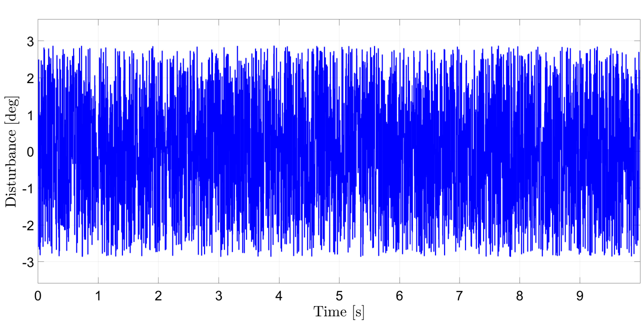

Fig. 4. Applied disturbance on each Euler angle.

A. Attitude Stabilization

Figs. 5 and 6 show the roll, pitch and yaw angles response and the control torque inputs corresponding to the attitude stabilization. the effectiveness of the proposed controller is obvious. The Euler angles converges to zero in finite-time. Moreover, the control inputs are chattering free with small values.

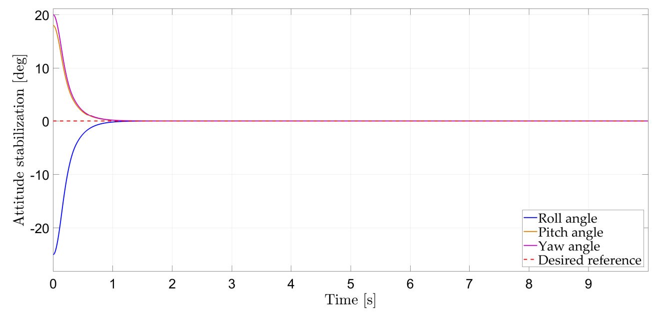

Fig. 5. Simulation results of attitude stabilization.

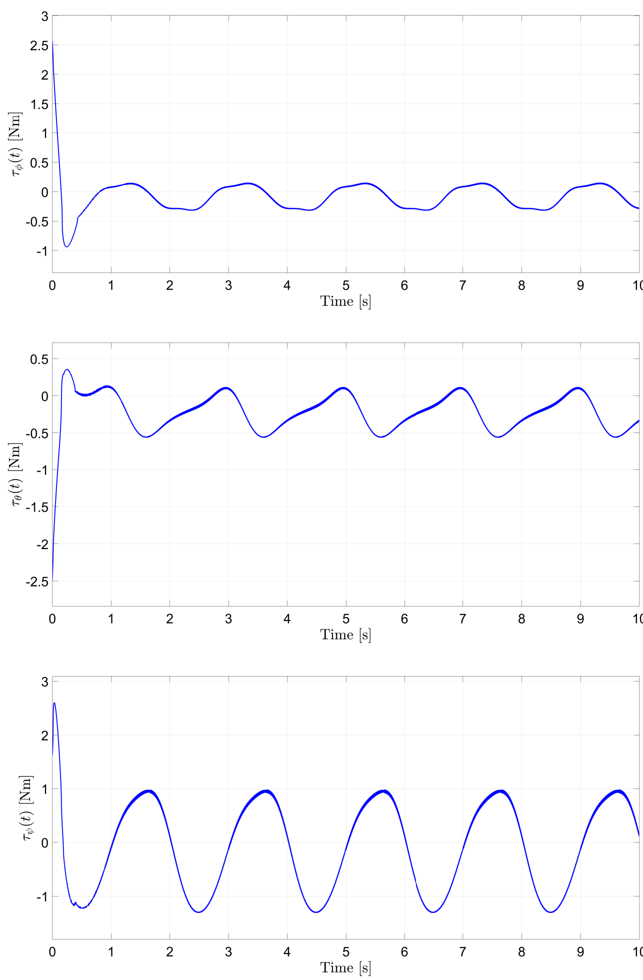

Fig. 6. Simulation results of control torque inputs.

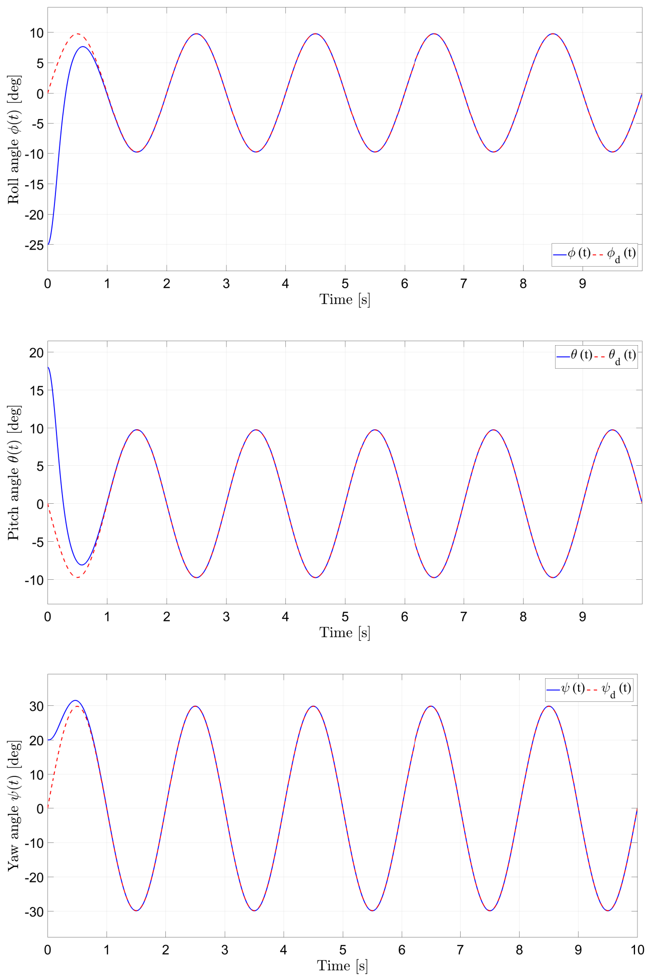

Fig. 7. Simulation results of attitude tracking.

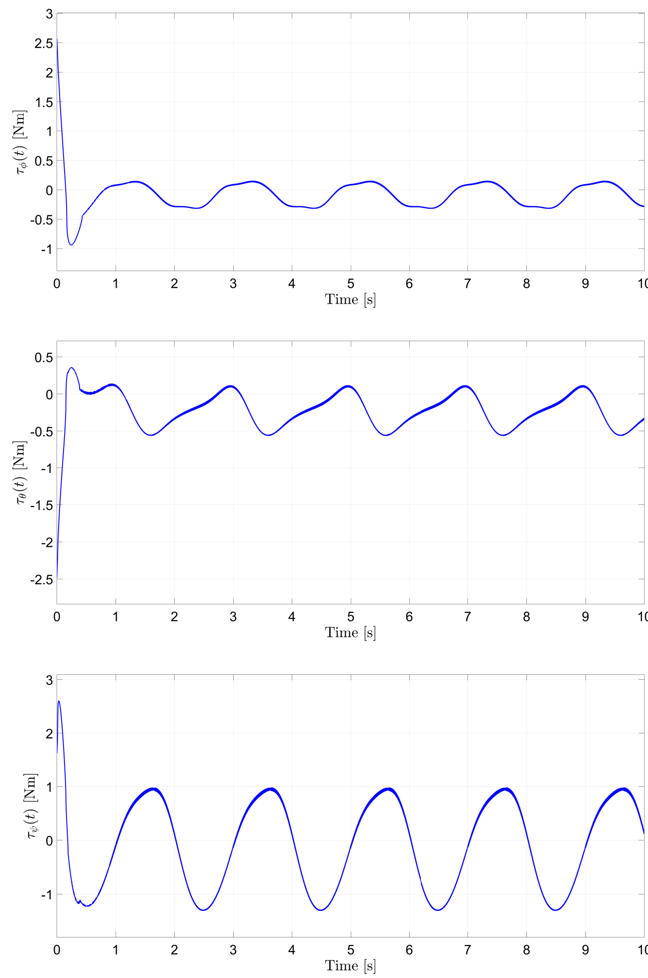

Fig. 9. Simulation results of control torque inputs.

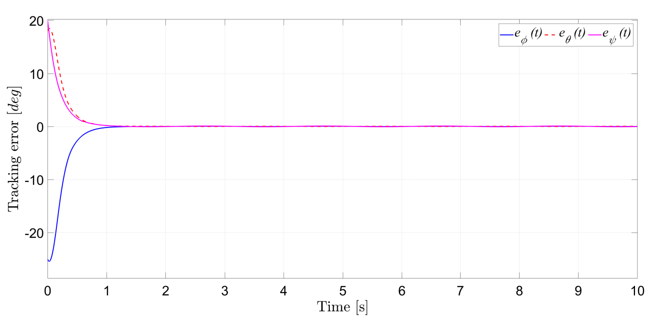

Fig. 8. Simulation results of attitude tracking error.

B. Attitude Tracking

Figs. 7, 8 and 9 show the roll, pitch and yaw angles attitude tracking, the attitude tracking error and the corresponding applied control torque inputs. The tracking performances are good. This can be seen in the attitude tracking error where the Euler angles error converge to zero in finite-time even in presence of disturbances.

V. Conclusions

In this paper, a robust sliding mode control combined with time delay estimation method has been designed and successfully simulated on a tri-rotor UAV system for the problem of attitude tracking and stabilization in presence of uncertainties and disturbances. The controller has been designed using TDE method to estimate in a simple way the uncertainties and to allow a small choice of switching gains in order to reduce chattering phenomenon and to ensure robustness. The obtained simulation results on the considered tri-rotor UAV system show clearly show the effectiveness of the proposed method in the attitude stabilization, attitude tracking and disturbance rejection.

Acknowledgements

The authors would like to thank to the Paraguayan Government for the financial through CONACYT research project (PINV15-0136) and the Programa de Vinculación de Científicos y Tecnólogos (PVCT 17-6).

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