Mixed Linear Programming Models for fruits and vegetables supply from family farms to rural schools as support for public policies[1]

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Abstract

The School Feeding Program of Paraguay (PAEP) has the function of providing food for children in elementary schools. Due to there are no efficient supply plans to aid local suppliers, we are interested in to minimize the purchase and transport costs and to maximize the local purchase subject to the school demand satisfaction in the benefited area. This study has identified sub-problems: the purchase of products from local family farms (pp), the vehicle routing problems (VRP) for multi-products picking (P) from the farms and multi-products delivery to schools (D). In this study have been designed four mathematical models based on Mixed Linear Programming: a three-stage model, two two-stage models, and a one-stage model. The experimental result (based on real data of a rural zone) shows that the multi-stage approaches use less computational time than the one-stage; however, the economic costs increase slightly.

Keywords: Local Government Supply Policies, Vehicle Routing Problem, Purchase Problem, Mixed Linear Programming.

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1. Introduction

The provision of fresh fruit and vegetables to schools in the interior of the country, within the framework of the PEAP [1] may not have the best supply processes management. The actual mechanisms could lead to dissatisfaction of demand, non-optimal allocation of suppliers, high logistical costs and even risks of failure to meet the objectives. In this way, according to information gathered, there are no records regarding the determination of the set of agricultural farms that can provide the set of schools in the benefited area, nor are there geo-referenced data about the distances between schools and farms. This paper addresses the problem raised to provide tools for decision making. First, the data mentioned above were collected, and then mathematical models were developed, which are of interest in this article.

From optimization this work focuses on the problem of determining the purchase and transport of fruit and vegetable products necessary for the elaboration of school menus established within the national government's policy. The initial analysis identifies three interconnected sub-problems: (P1) the optimal purchase of products from local family farms, (P2) the optimal routing of vehicles for the collection of multi-products from farms, and (P3) the optimal routing for the delivery of multi-products to schools.

Given the complexity it is necessary to develop efficient optimization models. Consequently, this paper proposes to study the problem considering four strategies based on Mixed Linear Programming (MILP) [2]: a three-phase strategy $P1 \rightarrow P2 \rightarrow P3$, two-phase strategies of two steps $P1 \oplus P2 \rightarrow P3$ and $P1 \rightarrow P2 \oplus P3$, and of a single phase $P1 \oplus P2 \oplus P3$. The note $P1 \oplus P2$ indicates that the problems $P1$ and $P2$ were approached together and therefore a single mathematical model is proposed, while $P1 \rightarrow P2$ means that the problem $P1$ was solved first and its results are data entry for the problem $P2$.

This work is organized as follows: section 2 presents the MILP models, section 3 carries out experimental tests and discussions of the proposed models, while section 4 presents conclusions and future work.

2. Proposed Models

Assumptions: A deposit (origin); a fleet of vehicles with different capacities; a vehicle can visit only once a subset of nodes (farms or schools); several types of products (multi-products); more than one vehicle can pass through a node for collection/delivery; split collection/delivery is allowed; local purchase is prioritized.

Índexes: i, j, s, r node index; q indicates product, and k indicates vehicle.

Constants: N number of nodes (farms, schools and product storage); K number of available vehicles; Q quantity of product types; $cext$ is cost of external purchase; d_{iq} is cost of external purchase q asked by the i-th school; d_q is the total amount of product q asked; o_{iq} is the quantity of the product q which provides the i-th farm; C_k is the capacity of k-th vehicle; cc_{iq} is the cost of purchasing a unit of the product q into i-th farm; ct_{ij} is the cost of road transport (i, j); and b_i indicates whether the i-th node is farm $(b_i = 1)$ or school $(b_i = 0)$.

Decision variables : $z_{iq} \in R^+$ is the product quantity q to purchase from the i-th farm; $z_{ijqk} \in R^+$ is the product quantity q to be transported by the k-th vehicle on the road (i, j); $zext_q \in R^+$ is the product quantity q to buy externally in order to meet demand; $x_{ijqk} \in R^+$ is the product quantity q into the k-th vehicle on the road (i, j); $y_{ijk} \in \{0, 1\}$ indicates whether the road (i,j) it is used by the vehicle k; whereas u_{ik} is an auxiliary variable for the elimination of sub-tours.

2.1 Strategy $P1 \rightarrow P2 \rightarrow P3$

This model addresses the problems detected independently in three phases. It should be noted that $P2$ and $P3$ are basically the Capacitive Vehicle Routing Problem (CVRP) but considering a divisible product whose basic model can be read at [2, 3, 4]. The CVRP model is applied in phases 2 and 3 in a similar approach to the VRP with Back-hauls (VRPB) [2]. In phase 2 the vehicles depart from the warehouse and load the products and return with the full load. The farms to be visited and products to be collected are determined in phase 1. In phase 3 the vehicles depart from the warehouse and unload at the schools.

For the first phase, the pp model is proposed for the problem $P1$.

Phase 1: **pp model** for $P1$

$$\text{Minimize } Z = \sum_{q=1}^{Q}\sum_{i=1}^{N} z_{iq} \cdot cc_{iq} + \sum_{q=1}^{Q} zext_q \cdot cext \tag{1}$$

s.t.

$$z_{iq} \leq o_{iq} \qquad \forall i; \forall q \tag{2}$$

$$\sum_{i=1}^{N} z_{iq} + zext_q = d_q \qquad \forall q \tag{3}$$

$$z_{iq} \geq 0 \qquad \forall i; \forall q \tag{4}$$

The objective function (1) minimizes the total cost of purchase, while the restriction (2) guarantees that the purchase does not exceed the offer of each farm and (3) ensures that the demand for each product is met. In the objective function, the inter purchase is prioritized when determining a high cost per external purchase, i.e. $cext >> cc_{iq}$.

2.2 Strategy $P1 \oplus P2 \rightarrow P3$

In this model, the sub-problems $P1$ and $P2$ are jointly solved in Phase 1. $P3$ is solved in Phase 2 as a CVRP [2]. For the problem, $P1 \oplus P2$, was proposed a mathematical model that combines the products purchase (pp) and the CVRP. In this scenario, a farm will be visited if a product is purchased; this differs from the classic CVRP problems where each node must be visited [4].

Phase 1: **ppCVRP model** for $P1 \oplus P2$

Minimize Z=

$$\sum_{q=1}^{Q}\sum_{i=1}^{N}\sum_{\substack{j=1\\j\neq i}}^{N}\sum_{k=1}^{K} z_{ijqk} \cdot cc_{iq} + \sum_{i=1}^{N}\sum_{\substack{j=1\\j\neq i}}^{N}\sum_{k=1}^{K} ct_{ij} \cdot y_{ijk} + \sum_{q=1}^{Q} zext_q \cdot cext \tag{5}$$

s.t.

$$\sum_{\substack{i=1\\i\neq j}}^{N}\sum_{k=1}^{K} y_{ijk} \leq 1; \qquad \forall j \qquad (6)$$

$$\sum_{\substack{i=1\\i\neq s}}^{N} y_{isk} - \sum_{\substack{j=1\\j\neq s}}^{N} y_{sjk} = 0; \qquad \forall s, \forall k \qquad (7)$$

$$u_{ik} - u_{jk} + N \cdot y_{ijk} \leq N - 1;\ u_1 = 1 \qquad \forall i \neq j \neq 1; \forall k \qquad (8)$$

$$2 \leq u_{ik} \leq N; \qquad \forall i, \forall k$$

$$\sum_{k=1}^{N} z_{ijqk} \leq o_{iq} \qquad \forall i; \forall j; \forall q \qquad (9)$$

$$\sum_{i=1}^{N}\sum_{\substack{i=1\\i\neq j}}^{N} z_{ijqk} + zext_q = d_q \qquad \forall q; \forall k \qquad (10)$$

$$\sum_{i=1}^{N}\sum_{\substack{i=1\\i\neq j}}^{N}\sum_{q=1}^{Q} z_{ijqk} \leq C_k \qquad \forall k \qquad (11)$$

$$z_{ijqk} \leq M \cdot y_{ijk} \qquad \forall i; \forall j; \forall q; \forall k \qquad (12)$$

$$z_{ijqk} \geq 0 \qquad \forall i, \forall j, \forall q, \forall k \qquad (13)$$

$$y_{ijk} \in \{0, 1\} \qquad \forall i, \forall j, \forall k \qquad (14)$$

The objective function (5) minimizes the total cost of purchasing and collecting products. With (6) it is indicated that a farm can be reached once by a vehicle, (7) and (8) represent the balance restrictions and sub-tour elimination respectively, (9) indicates that the purchase must not exceed the offer of each product on each farm, (10) represents the total satisfaction of demand, (11) is the vehicle's capacity restriction, (12) activates the cost of transporting the products.

2.3 Strategy $P1 \rightarrow P2 \oplus P3$

In this strategy, $P1$ is addressed in phase 1 using the same pp model presented in section 2.1. Phase 2 deals with VRP picking and multi-product delivery (VRPmPD). The proposed model differs from the classic VRP with Picking and Delivery, and also from VRP Back-haul [2].

Phase 2: **VRPmPD model** for $P2 \oplus P3$

$$\text{Minimize } Z = \sum_{i=1}^{N}\sum_{\substack{j=1\\j\neq i}}^{N}\sum_{k=1}^{N} y_{ijk} \cdot ct_{ij} \qquad (15)$$

s.t.

$$\sum_{\substack{i=1\\i\neq j}}^{N}\sum_{k=1}^{N} y_{ijk} \geq 1; \qquad \forall j \tag{16}$$

$$\sum_{\substack{i=1\\i\neq j}}^{N} y_{ijk} \leq 1; \qquad \forall j, \forall k \tag{17}$$

$$\sum_{\substack{i=1\\i\neq s}}^{N} y_{isk} - \sum_{\substack{j=1\\j\neq s}}^{N} y_{sjk} = 0; \qquad \forall s; \forall k \tag{18}$$

$$u_{ik} - u_{jk} + N \cdot y_{ijk} \leq N-1; u_1 = 1 \qquad \forall i \neq j \neq 1; \forall k \tag{19}$$

$$2 \leq u_{ik} \leq N \qquad \forall i \neq j$$

$$x_{ijqk} \leq M \cdot y_{ijk} \qquad \forall i, \forall j; \forall q, \forall k \tag{20}$$

$$\sum_{\substack{j=1\\j\neq i}}^{N}\sum_{q=1}^{Q} x_{ijqk} \leq C_k; \qquad \forall i, \forall k \tag{21}$$

$$\sum_{\substack{j=1\\j\neq i}}^{N}\sum_{k=1}^{K} x_{ijqk} - \sum_{\substack{r=1\\r\neq i}}^{N}\sum_{k=1}^{K} x_{riqk} = b_i \cdot o_{iq} + (1-b_i) \cdot d_{iq} \qquad \forall i, \forall q \tag{22}$$

$$(\sum_{\substack{j=1\\j\neq i}}^{N} x_{ijqk} - \sum_{\substack{r=1\\r\neq i}}^{N} x_{riqk})(1-b_i) \geq (\sum_{\substack{j=1\\j\neq i}}^{N} x_{ijqk} - \sum_{\substack{r=1\\r\neq i}}^{N} x_{riqk}) b_i \qquad \forall i; \forall q; \forall k \tag{23}$$

$$x_{ijqk} \geq 0 \qquad \forall i, \forall j, \forall q, \forall k \tag{24}$$

$$y_{ijk} \in \{0,1\} \qquad \forall i, \forall j, \forall k \tag{25}$$

In (15) we seek to minimize the total cost of routing. The restrictions (16) to (19) determine the conditions of a valid routing, where each node must be visited by at least one vehicle, while each vehicle must visit one node at most once (20) activates the cost of the product transport, (21) indicates that the total capacity of the vehicle after visiting the node i must not exceed maximum transport capacity. (22) ensures that the total quantity of product at the farm/school is collected/delivered. (23) determines that the vehicle load after visiting a farm/school should be higher/less than the entry load.

2.4 Strategy $P1 \oplus P2 \oplus P3$

In this strategy, the three sub-problems are tackled together in a single phase. For this purpose, a single mathematical model is proposed.

ppCVRPmPD model for $P1 \oplus P2 \oplus P3$

Minimize Z=

$$\sum_{\substack{i=1\\i\neq 1}}^{N}\sum_{q=1}^{Q}\sum_{k=1}^{N} x_{ijqk}\cdot cc_{iq} + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} y_{ijk}\cdot ct_{ij} + \sum_{j=1}^{N}\sum_{q=1}^{Q}\sum_{k=1}^{N} x_{1jqk}\cdot cext \tag{26}$$

s.t.

$$\sum_{\substack{i=1\\i\neq j}}^{N} y_{ijk} \leq 1 \qquad \forall j;\forall k \tag{27}$$

$$\sum_{\substack{i=1\\i\neq s}}^{N} y_{isk} - \sum_{\substack{j=1\\j\neq s}}^{N} y_{sjk} = 0 \qquad \forall s;\forall k \tag{28}$$

$$u_{ik} - u_{jk} + N\cdot y_{ijk} \leq N-1; u_1 = 1 \qquad \forall i\neq j\neq 1;\forall k$$

$$2 \leq u_{ik} \leq N \qquad \forall i\neq j \tag{29}$$

$$x_{ijqk} \leq M\cdot y_{ijk} \qquad \forall i,\forall j;\forall q;\forall k \tag{30}$$

$$x_{i1qk} \leq 0 \qquad \forall i;\forall q;\forall k \tag{31}$$

$$z_{ijqk} \geq 0 \qquad \forall i,\forall j,\forall q,\forall k \tag{32}$$

$$y_{ijk} \in \{0,1\} \qquad \forall i,\forall j,\forall k \tag{33}$$

The objective function (26) minimizes the total costs of purchasing and transporting products. (27) to (29) indicate the conditions of a valid routing. (30) is the activation of the transport cost. It should be noted that restrictions on loading products (22) to (23) used on the model $P1 \rightarrow P2 \oplus P3$ are applied entirely to this model.

3. Experiment

For this work, real data obtained from the Department of Caazapa, Paraguay, have been used [5]. The data of the geographic coordinates has allowed finding the matrix of distances and cost of transport (Table 1) [6]. Different instances of the whole set were taken. The computational tests were solved using IBM ILOG CPLEX software on a portable computer with 2.20 GHz Intel Core i3 processor with 4 Gb RAM. In Table 2, the results are presented; N indicates number of nodes and K number of vehicles. A small set of nodes (N) has been taken, with Q=3 types of products offered on the farms (onion, red pepper, and orange). The experiment consisted in testing three instances of a set of nodes (N=7, N=9, and N=11) and vehicles (K=2 and K=3), whose capacities (kg) are 250 kg (k=1); 300 kg (k=2) and 250 kg (k=3). These capacities were selected for experimental purposes.

As can be seen in Table 2, the cost suffers a slight reduction as the strategy groups match up the models to the ppCVRmPD model. In terms of computational solving time, this increases for the ppCVRmPD model. Notice that, for pp+CVRP+CVRP and ppCVRP+CVRP the number of nodes

Table 1: Transportation costs (USD)

	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10	n=11
n=1	0	0.0001	22.854	19.910	19.564	37.223	7.531	3.463	10.994	11.686	11.946
n=2	0.0001	0	22.854	19.910	19.564	37.223	7.531	3.463	10.994	11.686	11.946
n=3	22.854	22.854	0	2.943	3.290	12.379	16.360	20.257	19.824	20.516	20.776
n=4	19.910	19.910	2.943	0	0.346	9.436	13.408	17.313	16.880	17.573	17.833
n=5	19.564	19.564	3.290	0.346	0	9.609	13.080	16.967	16.534	17.227	17.486
n=6	37.223	37.223	12.379	9.436	9.609	0	18.727	22.680	22.161	22.940	23.113
n=7	7.531	7.531	16.360	13.408	13.080	18.727	0	7.531	3.463	10.994	11.686
n=8	3.463	3.463	20.257	17.313	16.967	22.680	7.531	0	4.934	3.636	4.328
n=9	10.994	10.994	19.824	16.880	16.534	22.161	3.463	4.934	0	8.397	9.089
n=10	11.686	11.686	20.516	17.573	17.227	22.940	10.994	3.636	8.397	0	6.492
n=11	11.946	11.946	20.776	17.833	17.486	23.113	11.686	4.328	9.089	6.492	0

Table 2: Costs (USD) and time (sec.) totals of the proposed models

Instance	pp+CVRP+CVRP	ppCVRP+CVRP	pp+VRPmPD	ppCVRPmPD
N=7	USD 222.81	USD 131.23	USD 194.70	USD 194.70
K=2	0.131 s	0.087 s	4.07 s	2.05 s
N=9	USD 275.14	USD 246.70	USD 247.02	USD 219.45
K=3	0.144 s	0.122 s	888.056 s	370.079 s
N=11	USD 316	USD 286.60	-	-
K=3	4 s	3 s	-	-

used in routing calculation is N/2 while for pp+VRPmPD and ppCVRPmPD is N. For this reason a multi-stage model is scalable in comparison with the one-stage model. In Table 2, considering N = 11, pp+VRPmPD and ppCVRPmPD, they can not find solution due to computational resources limitations. There is a trade-off between the quality of solution and performance of models, i.e. *the more compact the model, the more resources are necessary.*

As an illustration of how vehicle routing works, Figures 1 and 2 show how products are collected and then delivered to schools. The supply/demand of products of each node is indicated in the table. The values in the vehicles indicate the quantity transported by each product on each road.

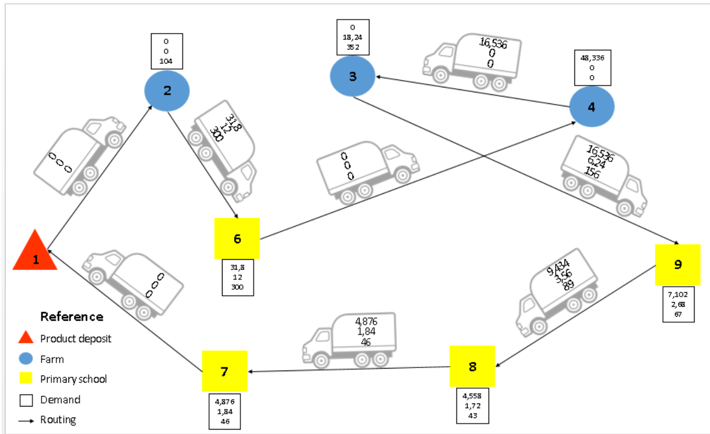

Figure 1: ppCVRmPD routing for vehicle 1 (k= 1)

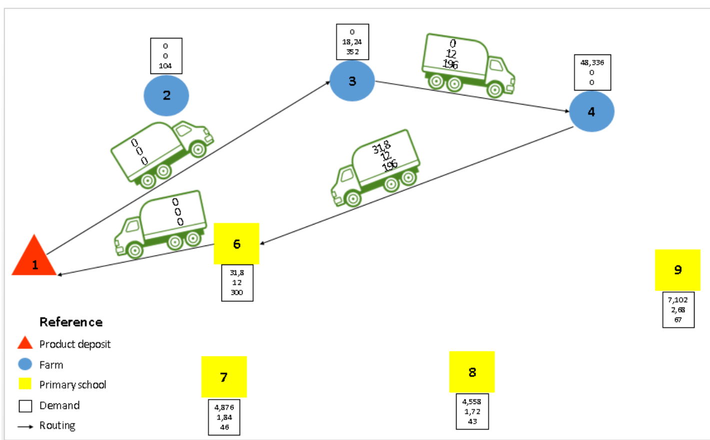

Figure 2: ppCVRmPD routing for vehicle 3 (k= 3)

Figure 1 illustrates routing for k=1, the same one goes to pick up 104 kg of product 3 (q=3) of farm 2 (n=2) and delivers it to the school 6 (n=6), then does a pickup by n=4 and n=3 with 16.536 kg for q=1; 6.24 kg (q=2) and 156 kg (q=156), leaving uncollected 31.8 kg (q=1) in n=4, with 12 kg (q=2), 196 kg (q=3) in n=3. Figure 2 shows how k=3 goes on to pick up n=3 and n=4 where k=1 not collected, 12 kg (q=2) and 192 kg (q=3) for delivery to school 6 (n=6). The model has determined not to use vehicle 2 (k=2).

4. Conclusion and Future Wok

In this work, mathematical models have been designed for the study problem. Sequential strategies, set and intermediate combinations were proposed. In addition, the joint approach ppCVRmPD achieves the best solution at the cost of more computational time. In this context, as the problem becomes more complex, the sequential approaches will be more attractive to generate solutions in reasonable times.

From a general point of view, it is possible to plan the decisions to purchase, collect and distribute the products (fruit and vegetables): the quantities to be purchased, where it would be most convenient to purchase, the number of vehicles needed, and which would be the optimal way to collect and distribute in order to reduce the costs involved in the process. It is worth mentioning that the model has been studied to be applied to other districts of the country, with some modifications or new restrictions that may appear, as well as consider time windows and multi-deposits, with experiments for more nodes.

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