

Environmental flows bifurcation simulation using FEniCS libraries

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ABSTRACT

The Pilcomayo River extends for about 1000 *km* from the Bolivian Andes to the Paraguay river [1]. The main factors that cause the peculiar behavior of the Pilcomayo River are: a) the important erosion of its riverbed; b) the progressive deposition of the carried sediments (about 125 million tons a year [1]), causing constant morphological change; c) the uniformity of the Chaco topography, which causes very low slopes that average around 0,03% [1].

In the above mentioned context, in order to regulate the distribution of the Pilcomayo waters between Paraguay and Argentina, a man-made bifurcation channel is maintained in the Chaco area.

This project is a first effort to achieve a better understanding of the behavior of the Pilcomayo River. The main objective at this step is the developement of a flow model of an incompressible, single-phase, non-stationary, viscous and isothermal fluid. We concentrate our efforts in the simulation of the bifurcation [2, 3]. For this particular case, we model the river flow using the Navier-Stokes equations for incompressible flow and an appropriate turbulence model. In our case we use the model $k - \varepsilon$ [4]. The equations of motion for the modeled fluid are:

$$\frac{D\vec{U}}{Dt} = -\frac{\nabla P}{\rho} + \nabla \left[\nu_{eff} \left(\nabla \vec{U} + (\nabla \vec{U})^T \right) \right] \quad (1)$$

$$\nabla \cdot \vec{U} = 0 \quad (2)$$

where $\vec{U}$ is velocity average of the fluid and P is the modified average pressure. $\nu_{eff} = \nu + \nu_t$ is the effective viscosity, in which ν is the molecular viscosity, depending on the properties of the fluid, and ν_t is the turbulent viscosity depending on the nature of the flow. The equation (1) is the Reynolds-Averaged Navier-Stokes equation (RANS) and the equation (2) is the average continuity equation, for incompressible flows.

The turbulent viscosity is given by:

$$\nu_t = C_\mu \frac{k^2}{\varepsilon} \quad (3)$$

where C_μ is a constant, k is the turbulent kinetic energy and ε is the dissipation ratio of the turbulent kinetic energy. k and ε are obtained from de transport equation for k and ε , respectively.

For the spatial discretization we apply the Finite Element Method and bilinear elements are used for velocity and pressure. For the stabilization of the numerical solution, the Streamline Upwind/Petrov Galerkin (SUPG) - Pressure Stabilizing/Petrov Galerkin (PSPG) method is applied [5].

The portability of the implementation is important, since future implementations are to consider more complex models giving adequate treatment to sediments deposition and morphological changes of the riverbed. The algorithm has been written in Python and for the implementation of the described methods we use FEniCS [6], a free software collection with extensive features for automated, efficient solution of differential equations. For the validation of the algorithm, it has been applied for solving the lid-driven cavity test, this being one of the usual benchmarks for the evaluation of Navier-Stokes solvers.

Quantitative comparison of the obtained results for the standard tests are satisfactory. The application of the developed model to the mentioned stretch of the Pilcomayo produces qualitative results, considering the complexity of the non-modeled additional factors of the represented phenomenon. The results are promising in order to have a better performance predictive model.

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