Hard and Easy Instances of L-Tromino Tilings [1]

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Outline

1. Introduction
   - Polyominoes
   - L-Tromino Tiling Problem
2. Tiling of the Aztec Rectangles
   - Aztec Rectangle
   - Aztec Rectangle with a single defect
   - Tiling Aztec Rectangle with unbounded number of defects
3. 180-Tromino Tiling
   - A rotation constraint
   - Forbidden Polyominoes

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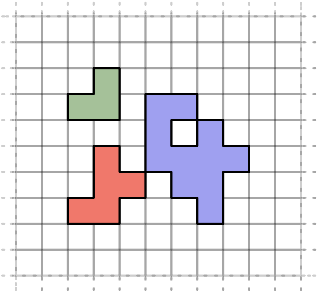

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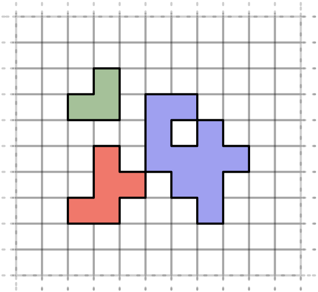

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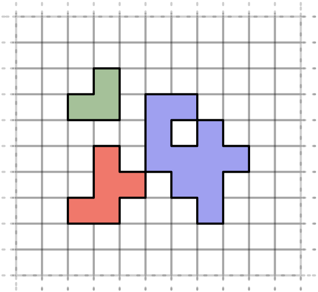

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- Two cell are adjacent if the Manhattan distance is 1.

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(a) A region R (b) A tiling of region R

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- T. Horiyama, T. Ito, K. Nakatsuka, A. Suzuki and R. Uehara (2012) constructed a one-one reduction from 1-in-3 SAT.

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Aztec Rectangle

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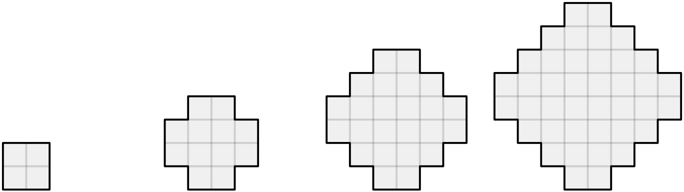

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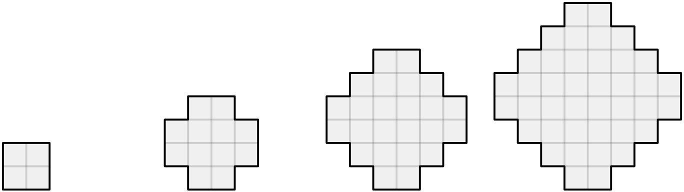

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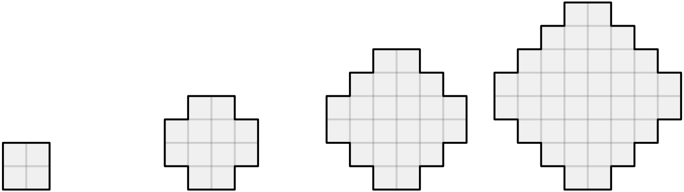

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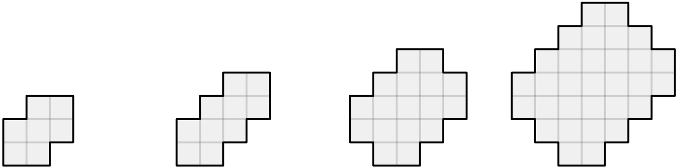

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Tiling Aztec Rectangle (cont'd)

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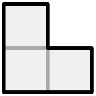
Each piece of L-tromino **covers 3 cells**.



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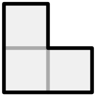

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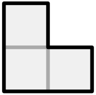

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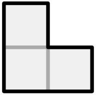

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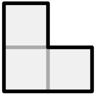

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Theorem

An Aztec rectangle $\mathcal{AR}_{a,b}$ has a tiling with L-trominoes
$\iff |\mathcal{AR}_{a,b}| \equiv 0 \pmod 3$
$\iff (a, b)$ *is equal to* $(3k, 3k')$ *or* $(3k-1, 3k'-1)$ *for some* $k, k' \in \mathbb{N}$.

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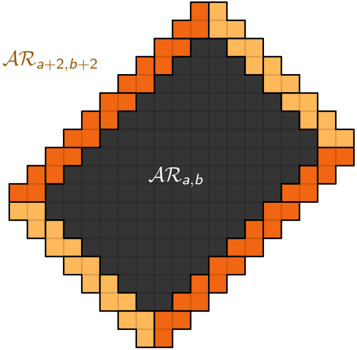

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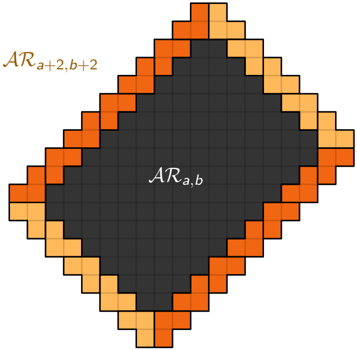

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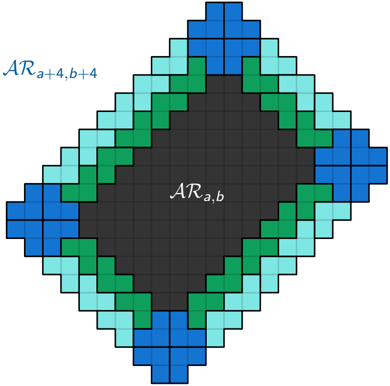

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- If (a, b) equals $(3, 3k')$, use pattern 3.
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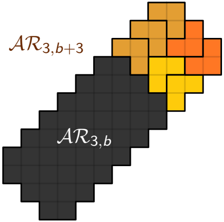

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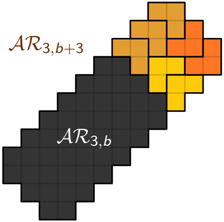

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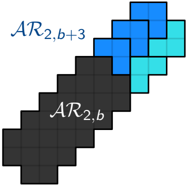

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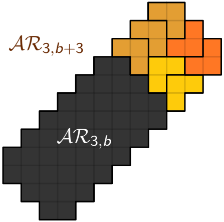

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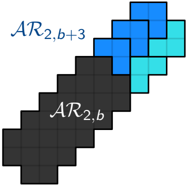

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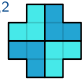
Base case: $\mathcal{AR}_{2,2}$ and $\mathcal{AR}_{3,3}$.



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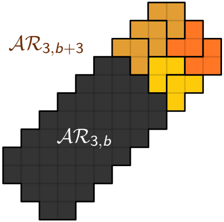

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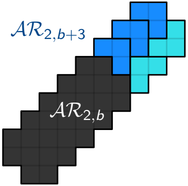

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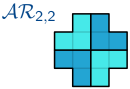

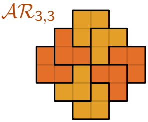

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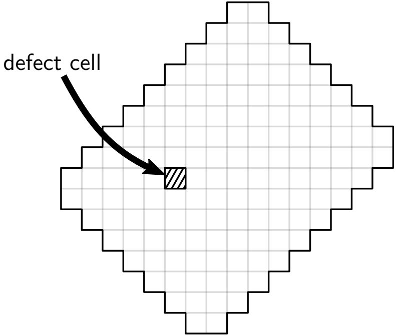

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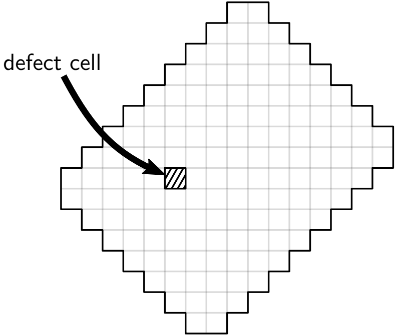

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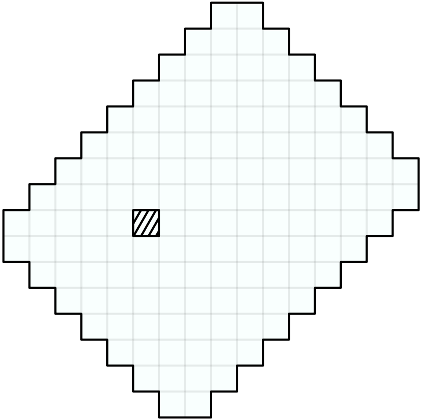

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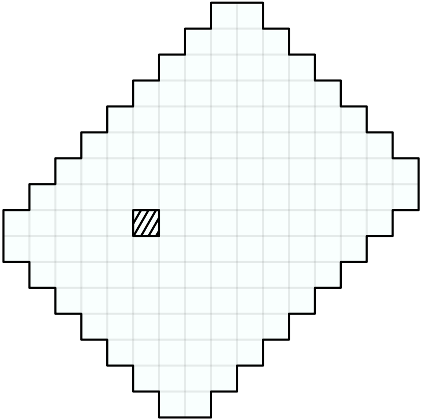
- Place a *fringe* where it covers the defect.



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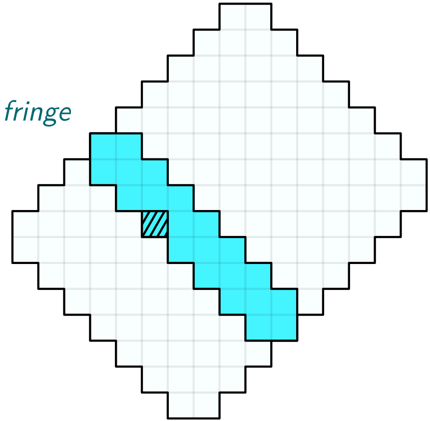

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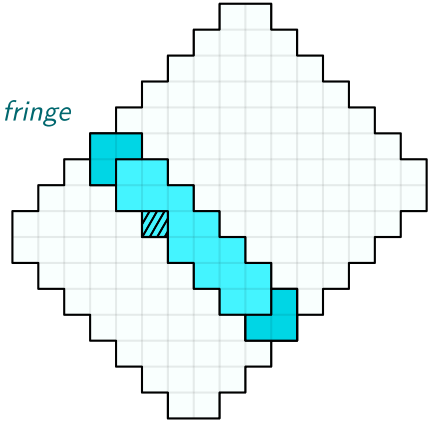

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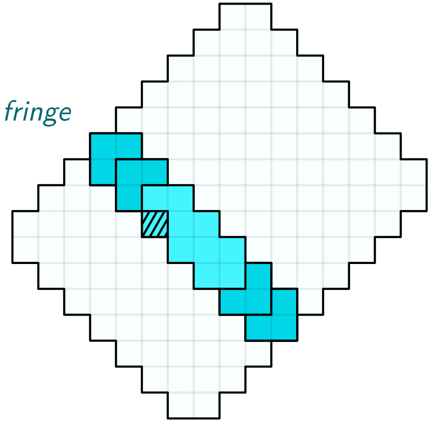

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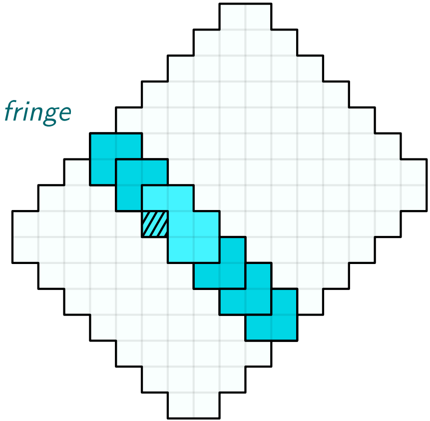

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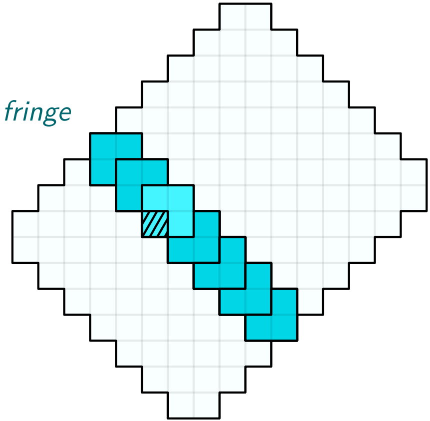

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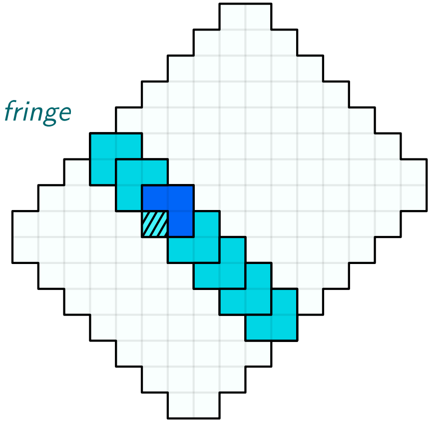

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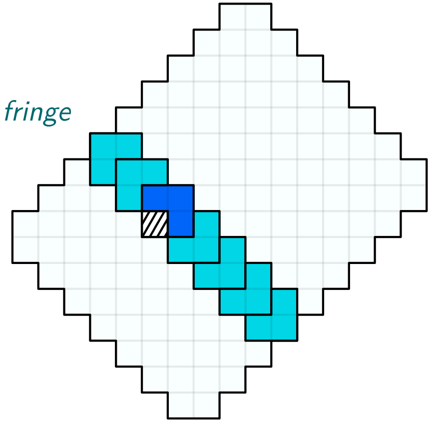

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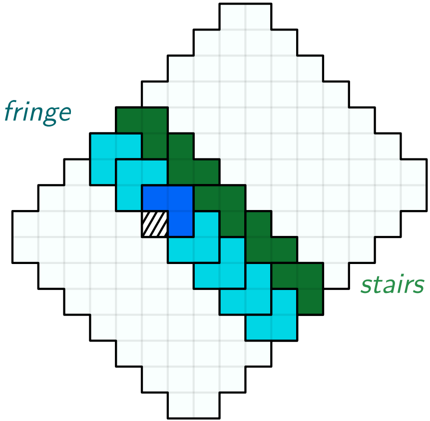

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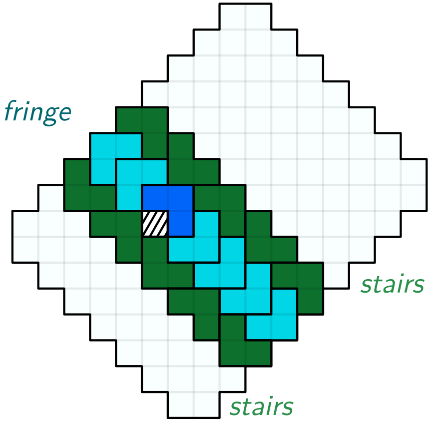

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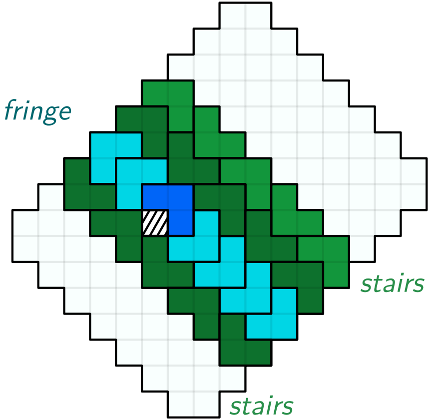

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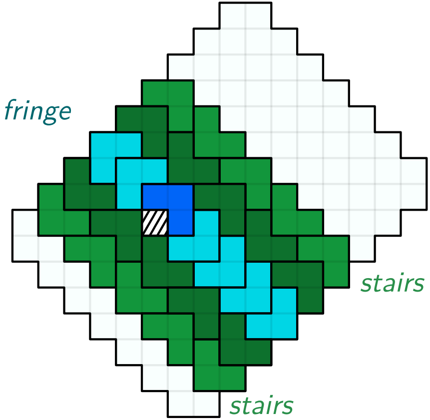

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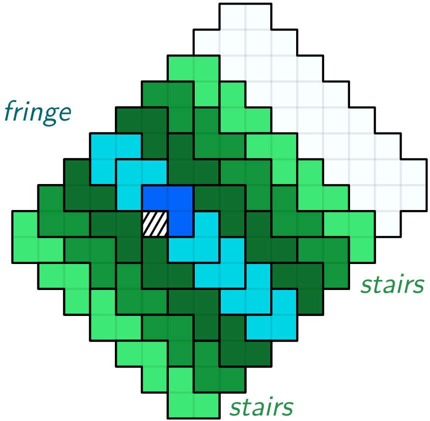

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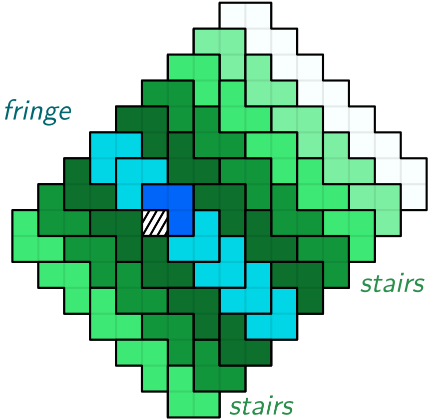

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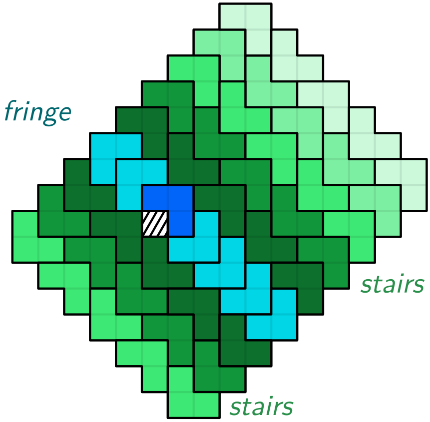

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3. 180-Tromino Tiling
   - A rotation constraint
   - Forbidden Polyominoes

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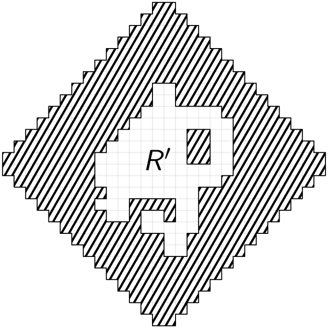

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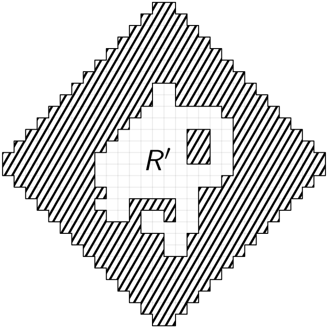

Theorem

*The problem of tiling Aztec Rectangle $\mathcal{AR}_{a,b}$ with an unbounded number of defects is **NP-complete**.*

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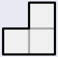
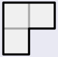
$\Sigma = \{$ **right-oriented** 180-trominoes $\} = \{$ ,  $\}$

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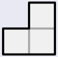
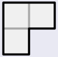
$\Sigma = \{$ **right-oriented** 180-trominoes $\} = \{$ ,  $\}$

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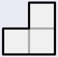
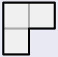
$\Sigma = \{$ **right-oriented** 180-trominoes $\} = \{$ ,  $\}$

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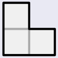
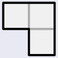
$\Sigma = \{$ **left-oriented** 180-trominoes $\} = \{$ ,  $\}$.

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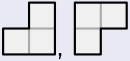
$\Sigma = \{$ **right-oriented** 180-trominoes $\} = \{$  $\}$

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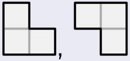
$\Sigma = \{$ **left-oriented** 180-trominoes $\} = \{$  $\}$.

With no loss of generality, we will only consider **right-oriented 180-trominoes**.

180°L-Tromino Tiling (cont'd)

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Theorem

There is a one-one correspondence between 180-tromino tiling and the triangular trihex tiling [Conway and Lagarias, (1990)].

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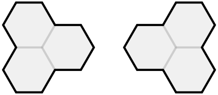

Two **triangular trihex**.

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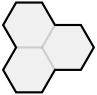

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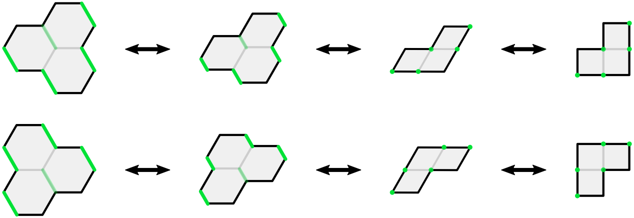

Transformation from **triangular trihex** to **180-tromino**

180°L-Tromino Tiling (cont'd)

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A **cell tetrasection** is a division of a cell into 4 equal size cells.

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A **tetrasected polyomino** $P^{\boxplus}$ is obtained by tetrasecting each cell of a poylomino P.

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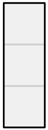

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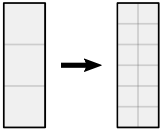

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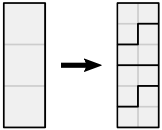

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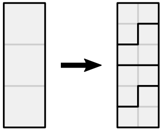

However, it is not know if the converse statement is true or false.

180°L-Tromino Tiling (cont'd)

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1-in-3 SAT $\leq_P$ I-tromino Tiling

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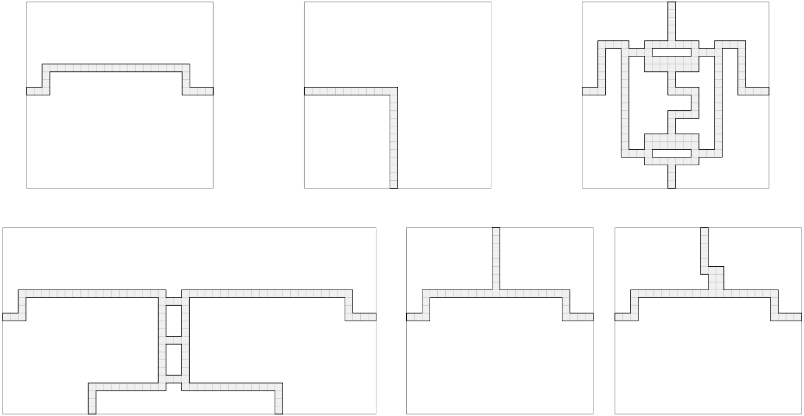

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In each gadget G, I-tromino tiling for G can be simulated with 180-tromino tiling for $G^{\boxplus}$.

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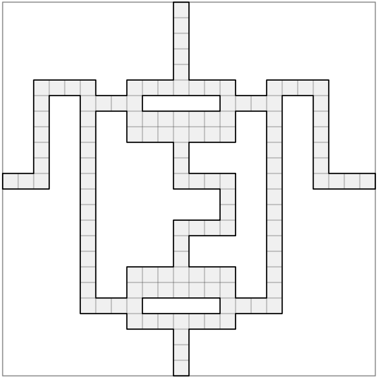

(a) Original gadget G.

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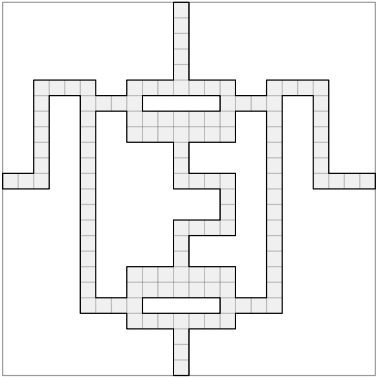

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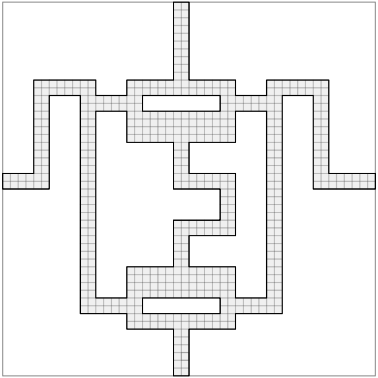

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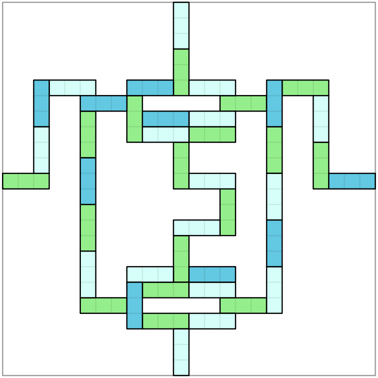

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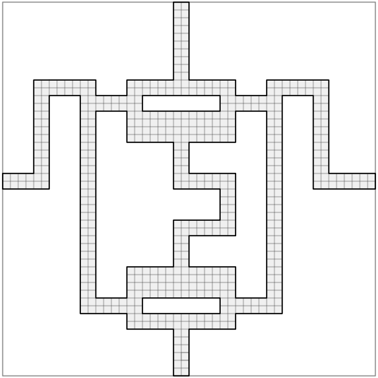

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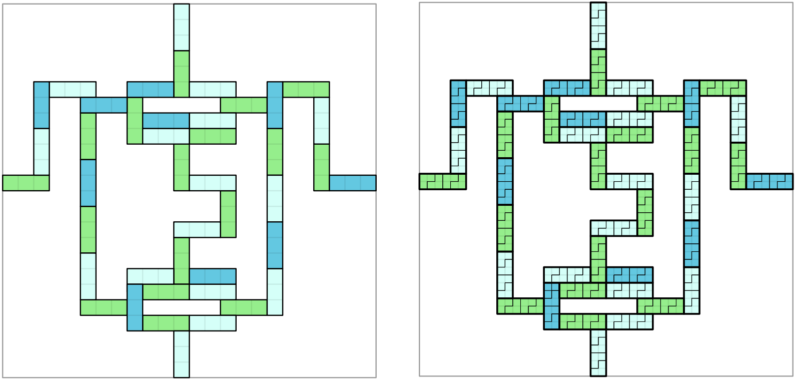

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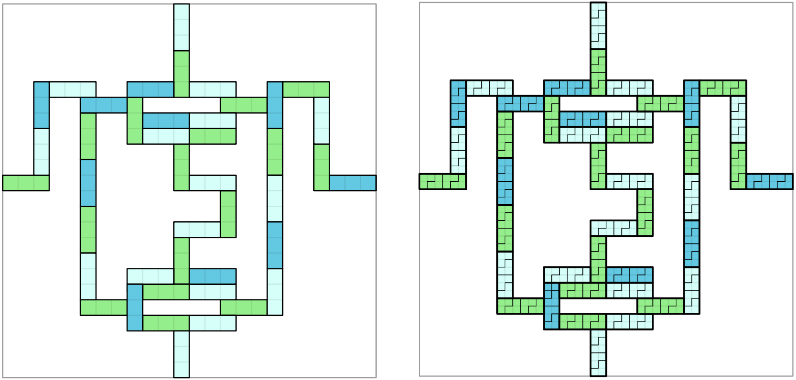

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Theorem

*180-tromino tiling is **NP-complete**.*

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Forbidden Polyominoes

The 180-tromino tiling can also be reduced to the **Maximum Independent Set** problem.

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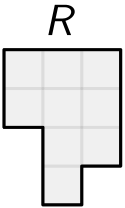

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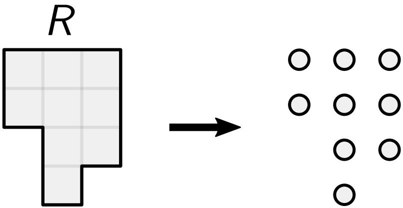

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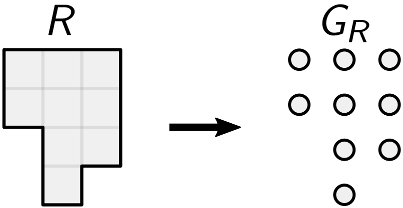

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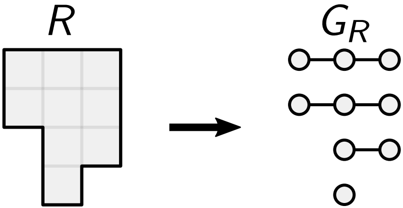

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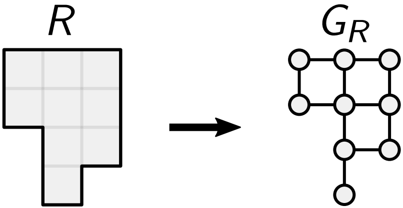

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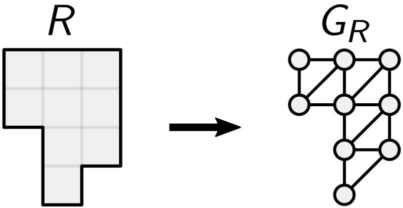

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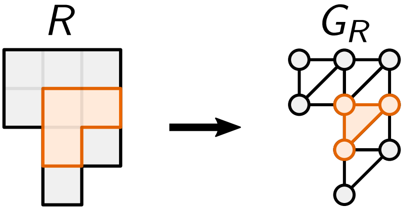

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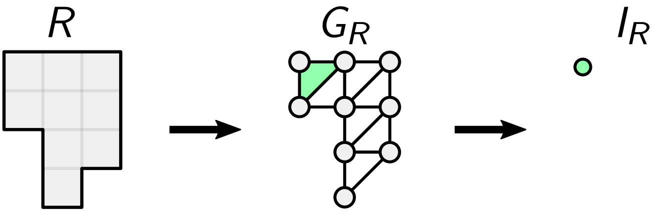

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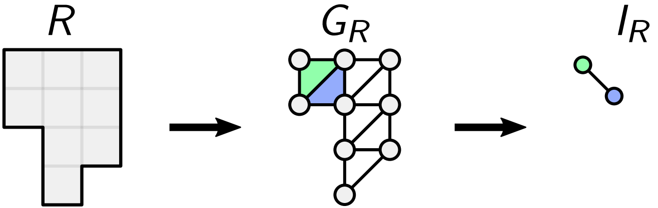

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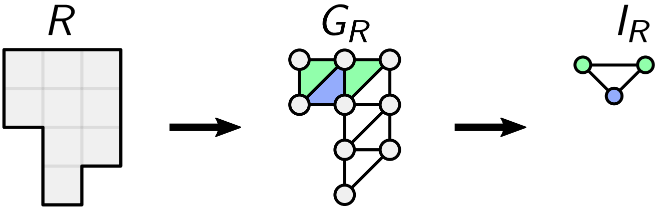

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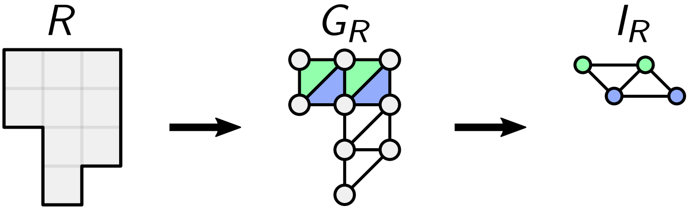

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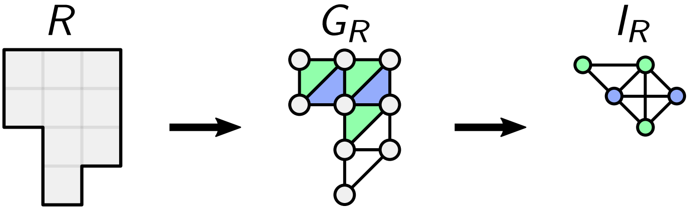

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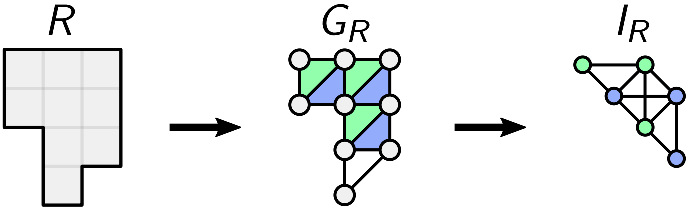

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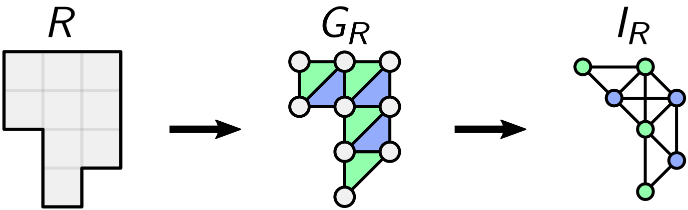

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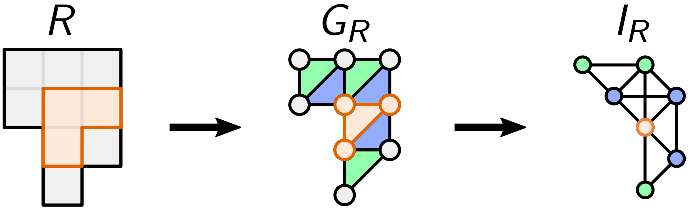

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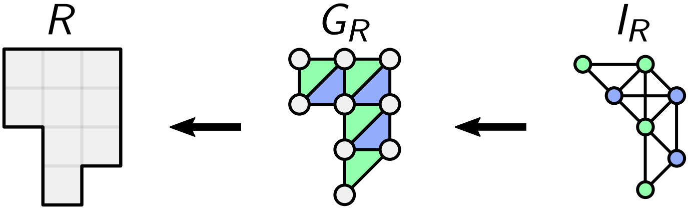
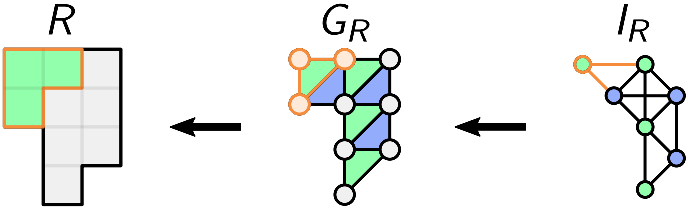
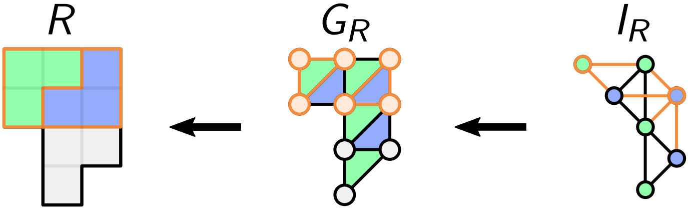
  - Add an edge where 3-cycles intersects.

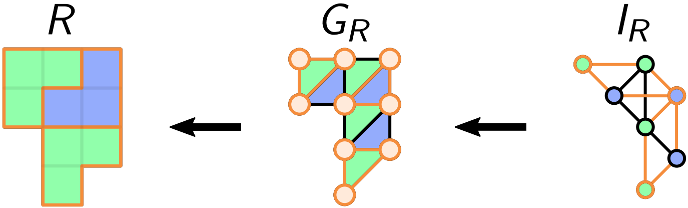
Forbidden Polyominoes (cont'd)



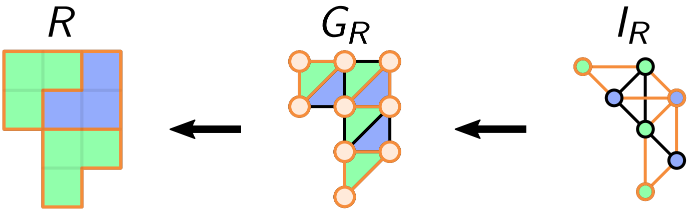
Forbidden Polyominoes (cont'd)




R
G_R
I_R


R
G_R
I_R

Forbidden Polyominoes (cont'd)



Theorem

Maximum Independent Set of I_R *is equal to* $\frac{|R|}{3}$
$\iff$ *R has a 180-tromino tiling .*

where $|R|$ the number of cells in a region R.

Forbidden Polyominoes (cont'd)

Forbidden Polyominoes (cont'd)

If I_G is **claw-free**, i.e., does not contain a claw as induced graph, then computing Maximum Independent Set can be computed in polynomial time.

Forbidden Polyominoes (cont'd)

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The following five polyominoes generates a distinct I_G with a **claw** in it.

Forbidden Polyominoes (cont'd)

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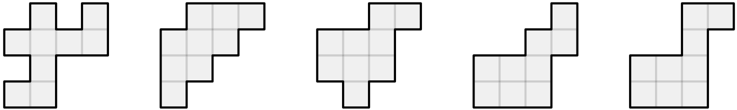
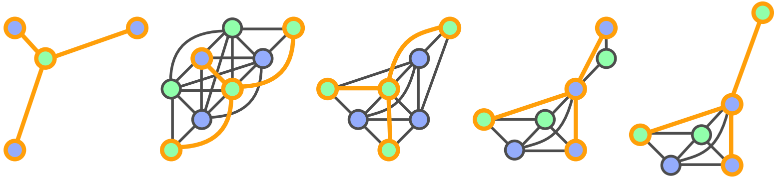
The following five polyominoes generates a distinct I_G with a **claw** in it.

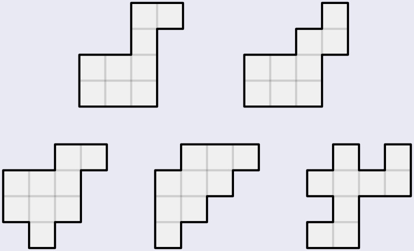




Forbidden Polyominoes (cont'd)

Theorem

*If a region R **doesn't** contains a rotated, reflected or sheared **forbidden polyomino**, then 180-tromino tiling can be computed in a polynomial time.*



Thank you!

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Thank you!

You can try the tetrasected cell tiling program in your phone browser: http://bit.ly/TetrasectedTiling